

# Central Control over Distributed Asynchronous Systems: A Tutorial on Software-Defined Networks and Consistent Network Updates

Klaus-T. Foerster



## Brief Preamble

- Focus on algorithmic/complexity issues in consistent updates in Software Defined Networks (SDNs)
  - Not so much on system etc. issues respectively SDNs themselves
- Two “bigger” connections to classic distributed computing halfway-in
  - Proof Labeling Schemes
  - Distributed Control Plane

## Network Updates

- The Internet: Designed for selfish participants
  - Often inefficient (low utilization of links), but robust
  
- But what if eg the Wide-Area Network is controlled by a single entity?
  - Examples: Microsoft & Amazon & Google ...
  - They spend hundreds of millions of dollars per year

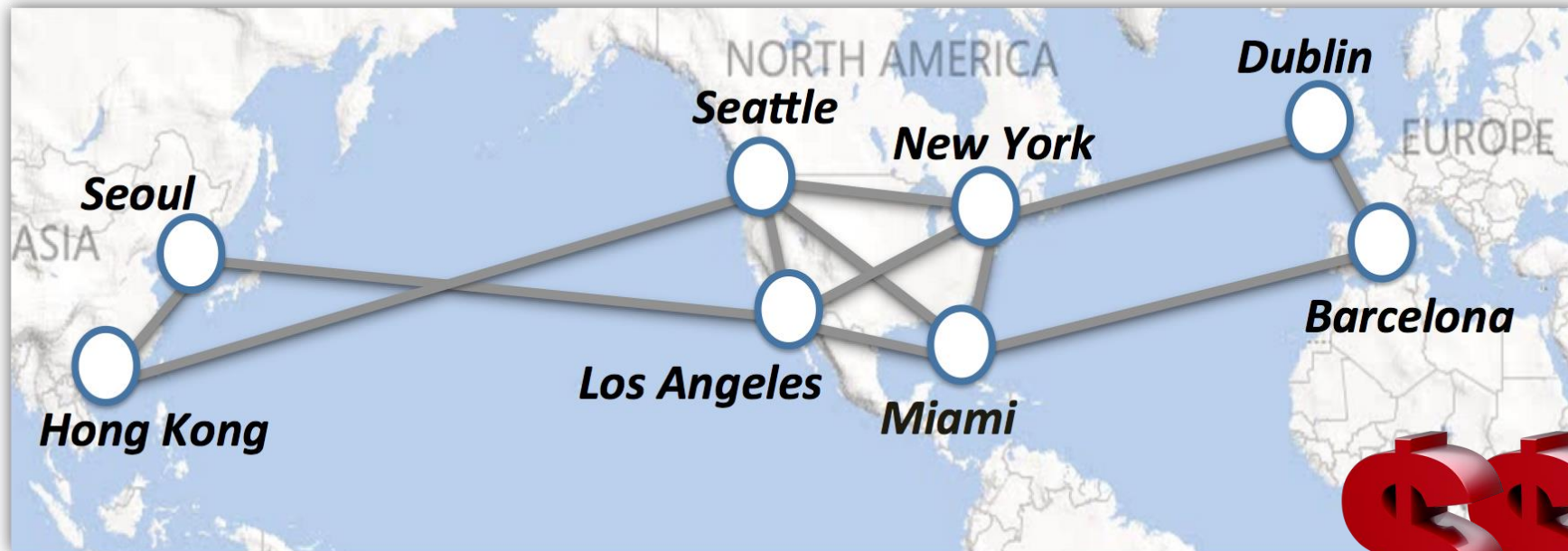




Also relevant in eg Data Center  
Networks, for ISPs etc

## Network Updates

Eg update link capacity at runtime?\*



Think: Google, Amazon, Microsoft

\*:RADWAN: Rate Adaptive Wide Area Network. R. Singh, M. Ghobadi, K.-T. Foerster, M. Filer, P. Gill. ACM SIGCOMM 2018

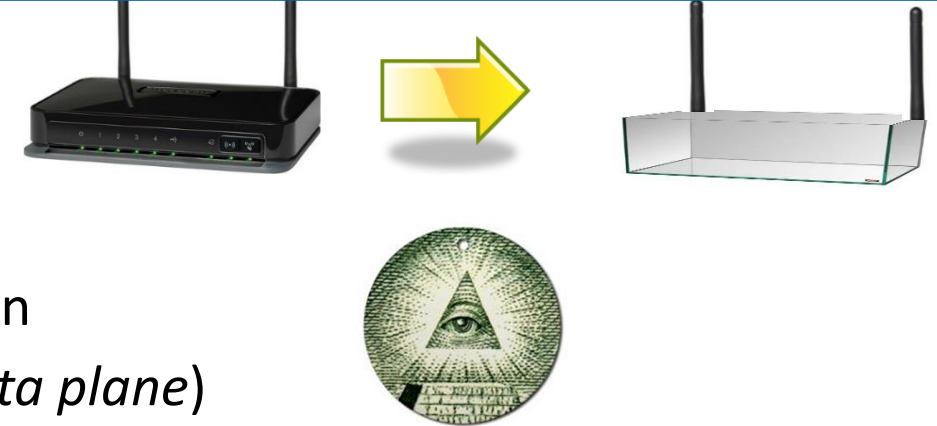
Note: There is also a lot of (prior) research on consistency before SDNs – can't cover everything in this tutorial

## Software-Defined Networking

- Possible solution:
  - **Software-Defined Networking (SDNs)**
- General Idea: Separate data & control plane in a network
- Centralized controller updates networks rules for optimization
  - Controller (*control plane*) updates the switches/routers (*data plane*)

See history section in:

*Survey of Consistent Software-Defined Network Updates*  
Klaus-Tycho Foerster, Stefan Schmid, Stefano Vissicchio  
*IEEE Communications Surveys & Tutorials*, 21(2), 2019



Virtual Services

Controller

Physical Network

- Logically centralized controller (eg implemented with replication)



*old* network  
rules



network updates



*new* network  
rules



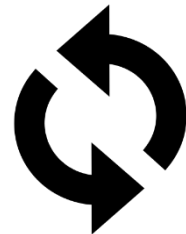
*old* network  
rules



network updates



*new* network  
rules





*old* network  
rules



*new* network  
rules

possible solution: be fast!



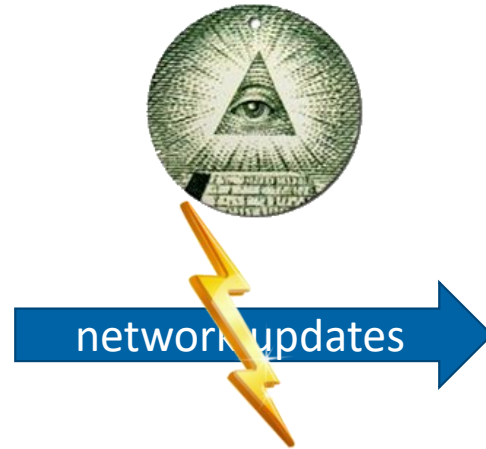
e.g., B4 (Google, 2013)

But they deviated from that a  
bit in the B4 2018 version...





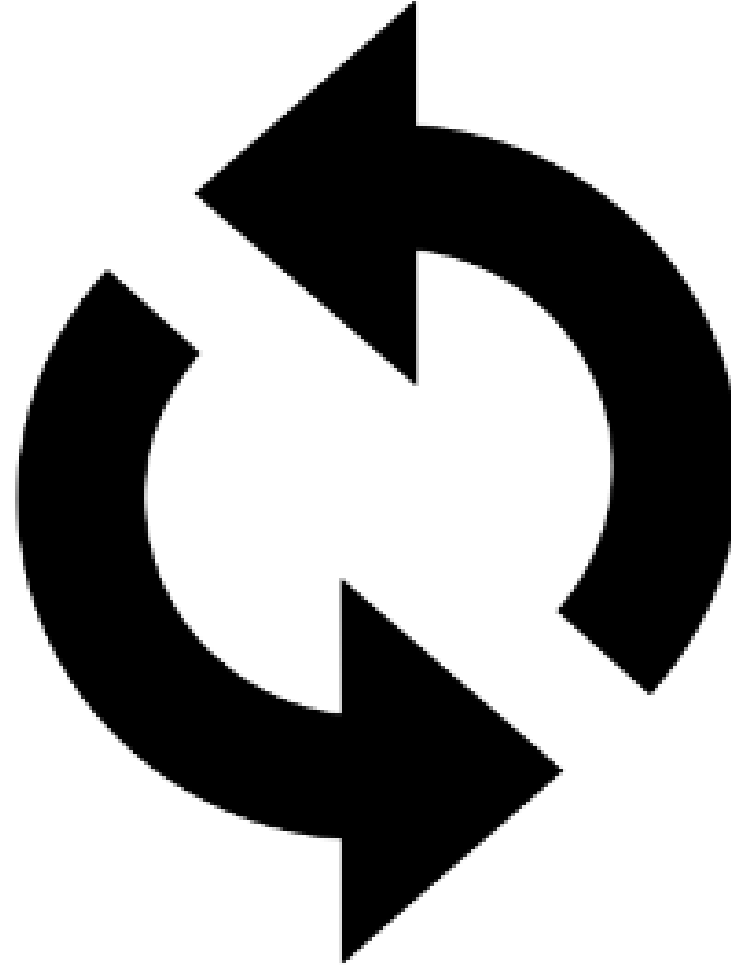
*old* network  
rules



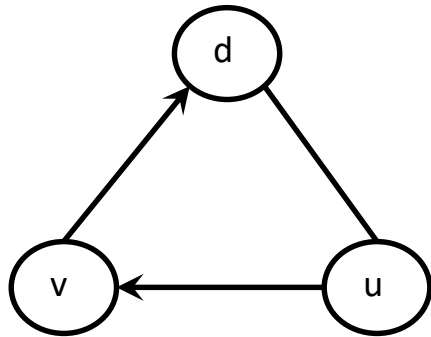
*new* network  
rules

Alternative: Be consistent!

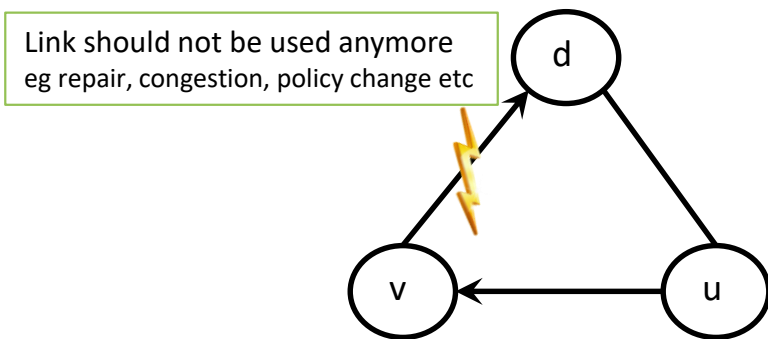
- Algorithms with guarantees



## Toy Example

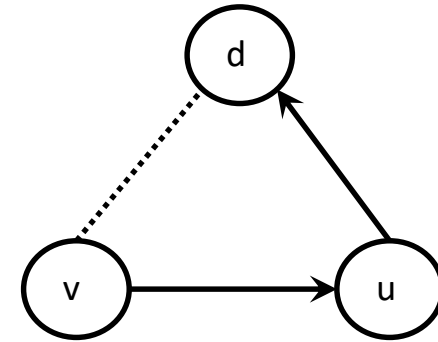
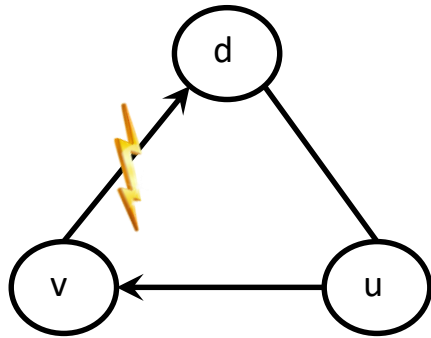


## Toy Example





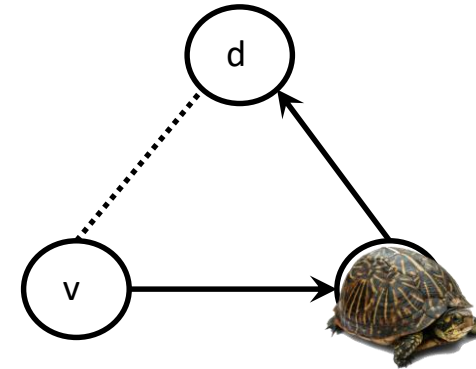
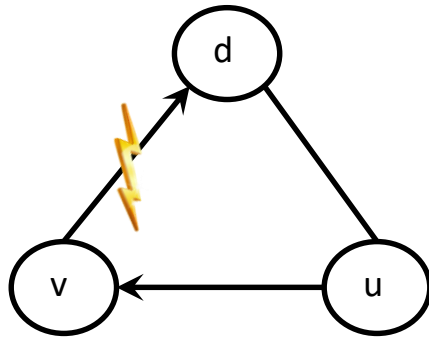
## Toy Example



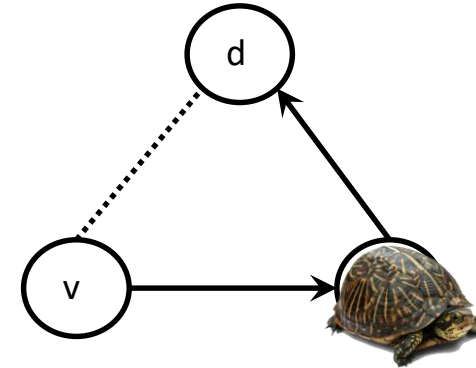
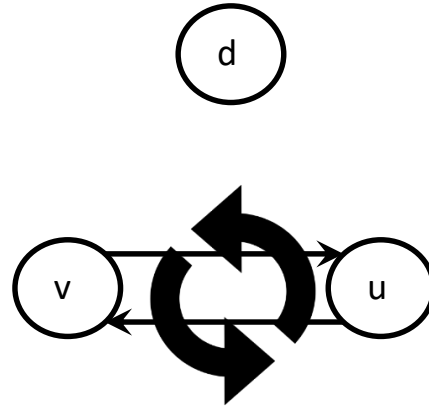
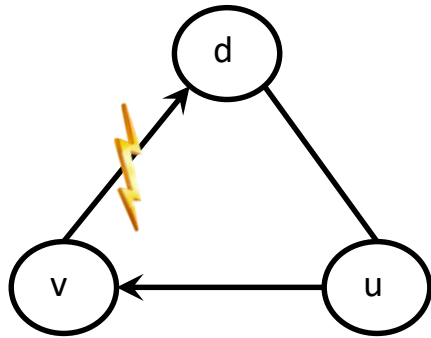
## Toy Example



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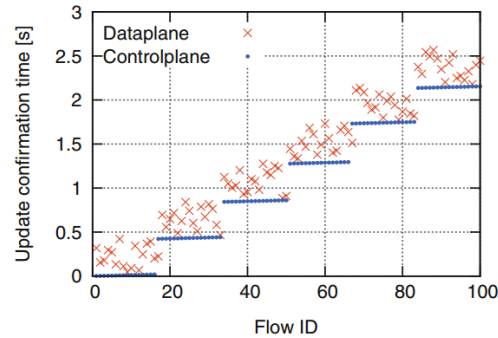


## Toy Example

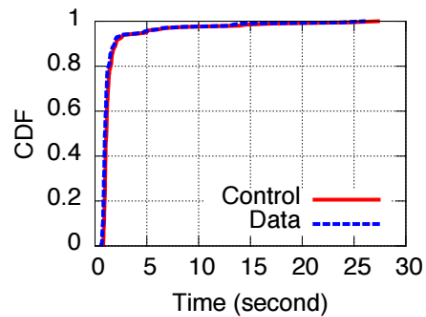




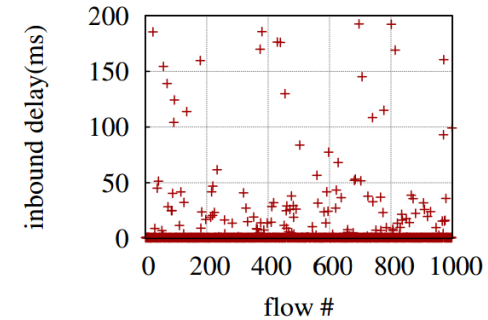
## Appears in Practice



*“Data plane **updates may fall behind** the control plane acknowledgments and may be even **reordered**.”*  
Kuzniar et al., PAM 2015

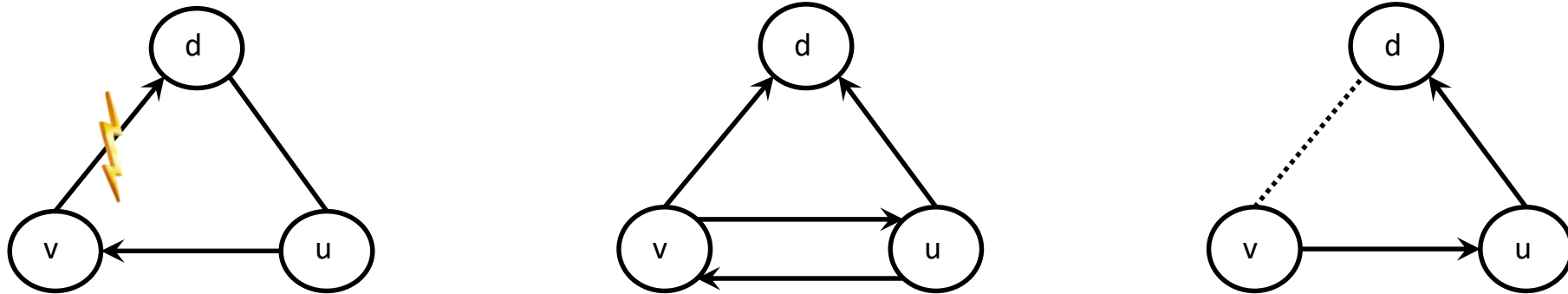


*“some switches can **‘straggle,’** taking substantially **more time** than average (e.g., **10-100x**) to apply an update”*  
Jin et al., SIGCOMM 2014



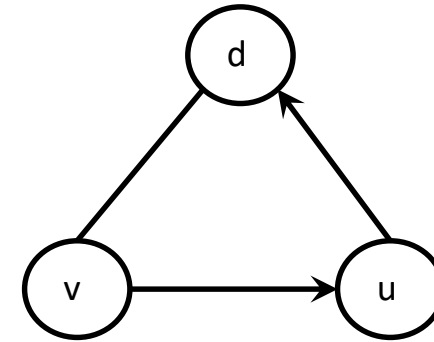
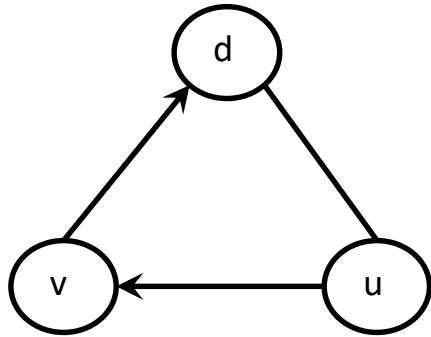
*“...the inbound latency is **quite variable** with a [...] standard deviation of **31.34ms**...”*  
He et al., SOSR 2015

## Toy Example

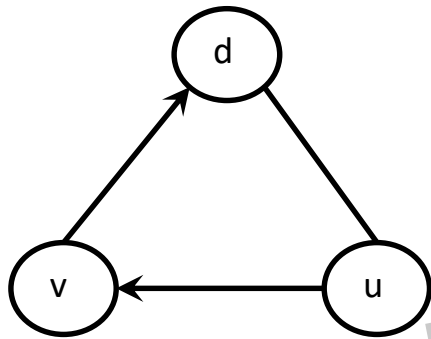


Old and new states exist simultaneously in a limbo state

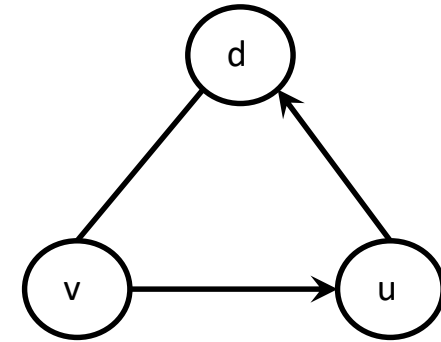
## Ordering Solution: Go backwards through the new routing tree



## Ordering Solution: Go backwards through the new routing tree

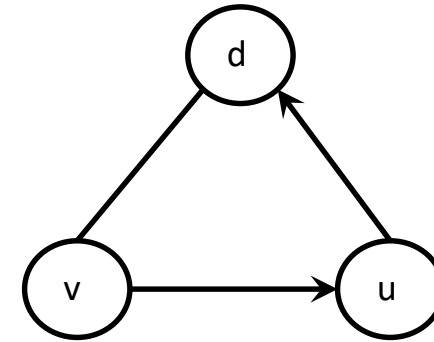
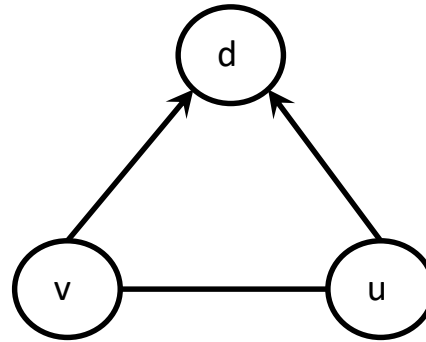
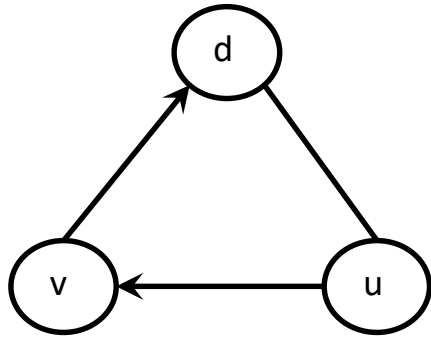


**Update!**

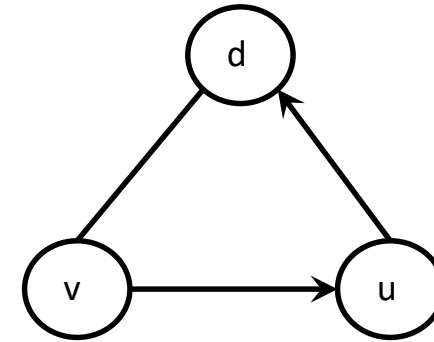
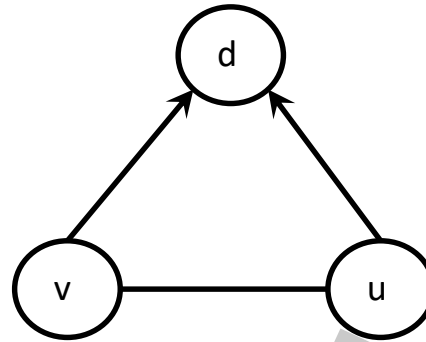
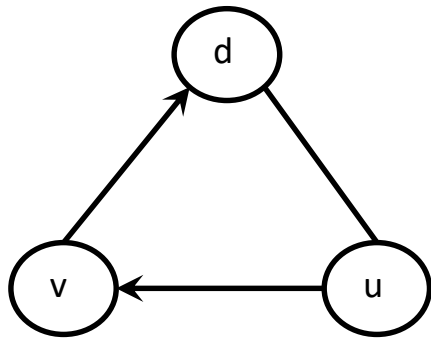




## Ordering Solution: Go backwards through the new routing tree



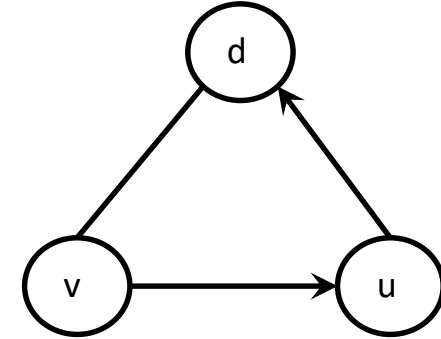
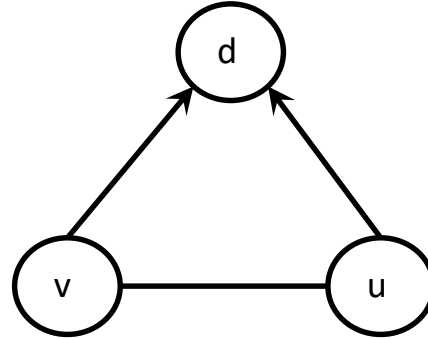
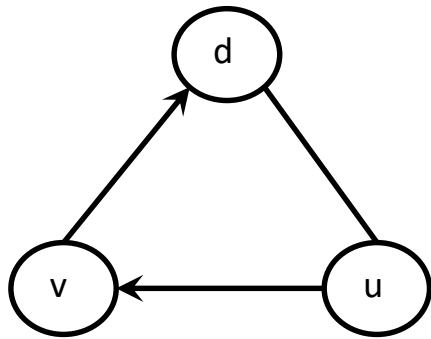
## Ordering Solution: Go backwards through the new routing tree



Ack!



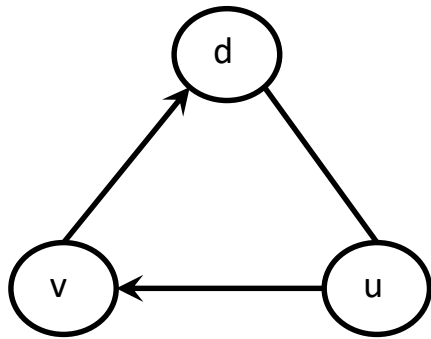
## Ordering Solution: Go backwards through the new routing tree



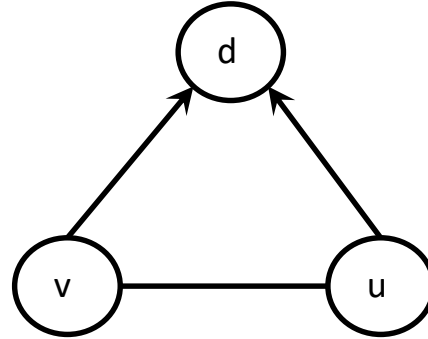
**Update!**



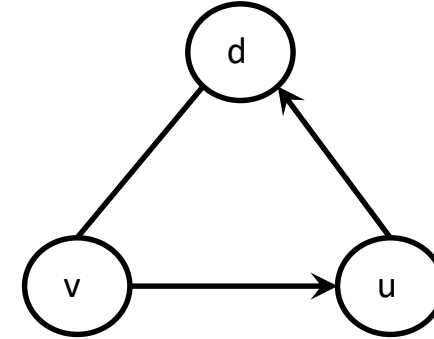
## Ordering Solution: Go backwards through the new routing tree



Round 0 (*old*)



Round 1



Round 2 (*new*)

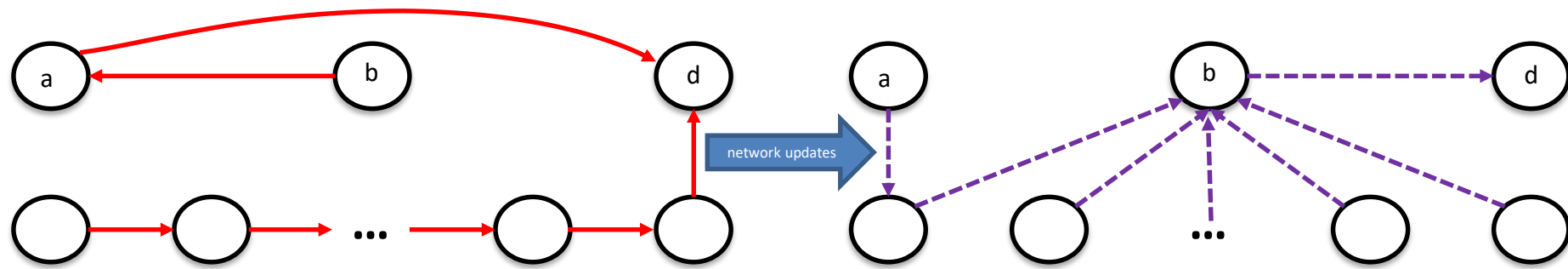
- Always works for single-destination rules
  - Also for multi-destination with sufficient memory (“split”)
- Schedule length: tree depth (up to  $\Omega(n)$ )
  - Optimal scheduling algorithms?

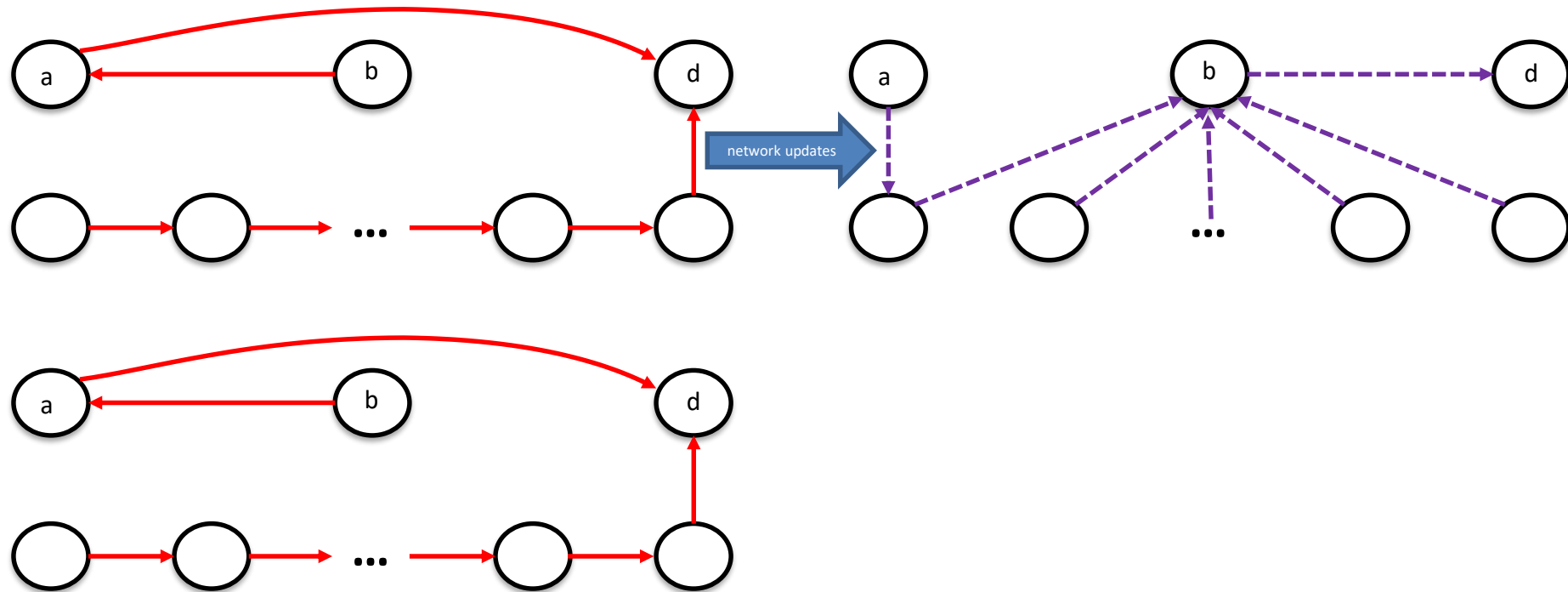
More on scheduling multiple policies:  
Basta et al: Efficient Loop-Free Rerouting of  
Multiple SDN Flows. ToN 2018

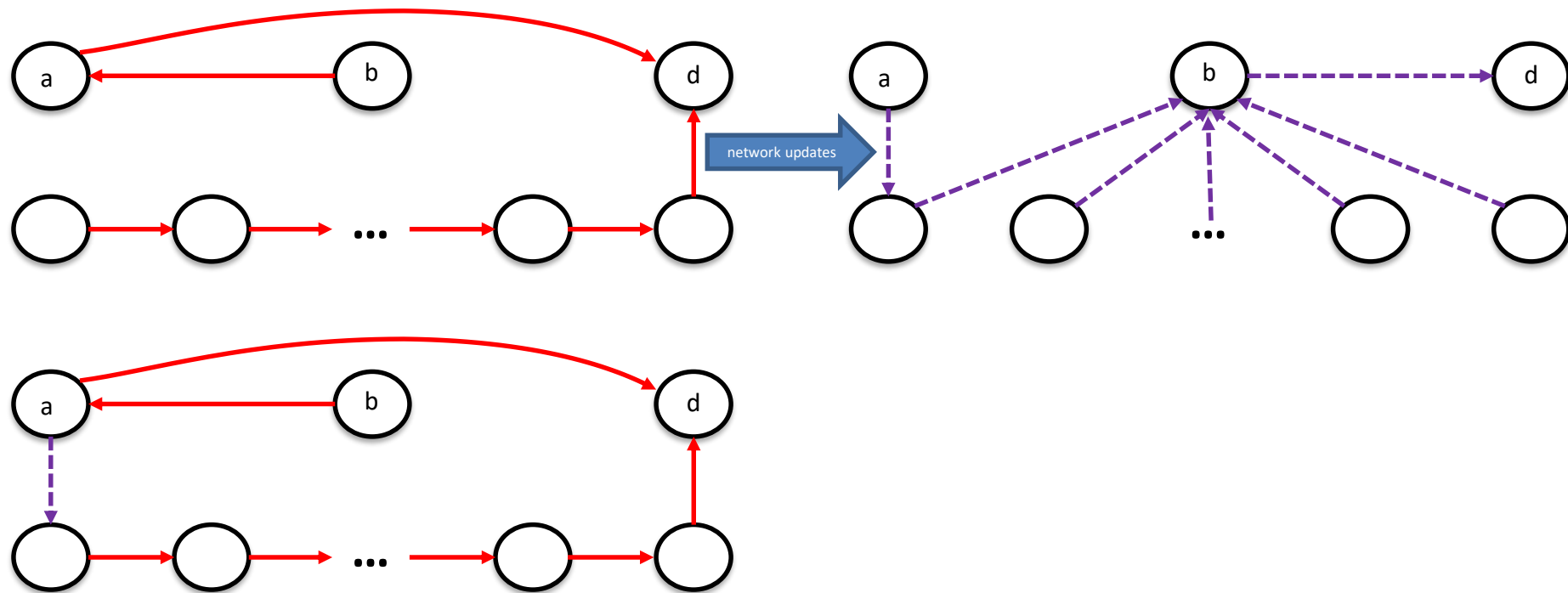


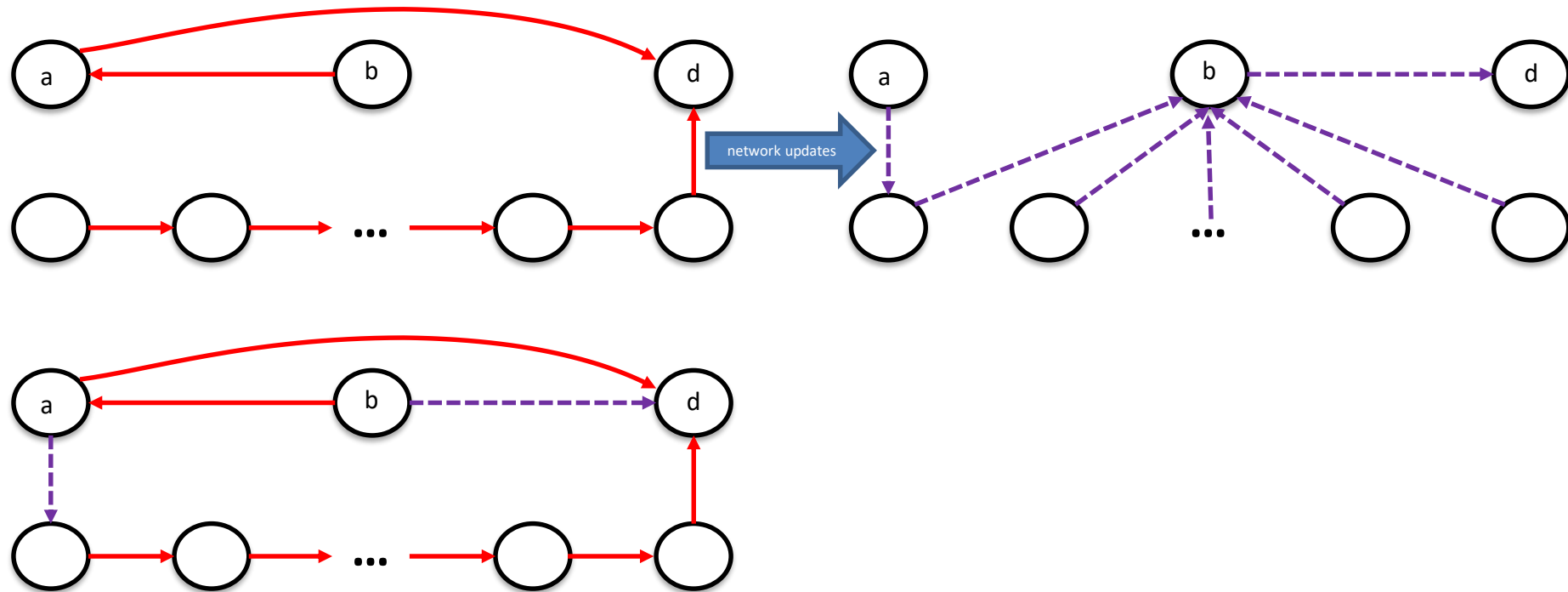
## Greedy? Update as many as possible per round

- Always works 😊

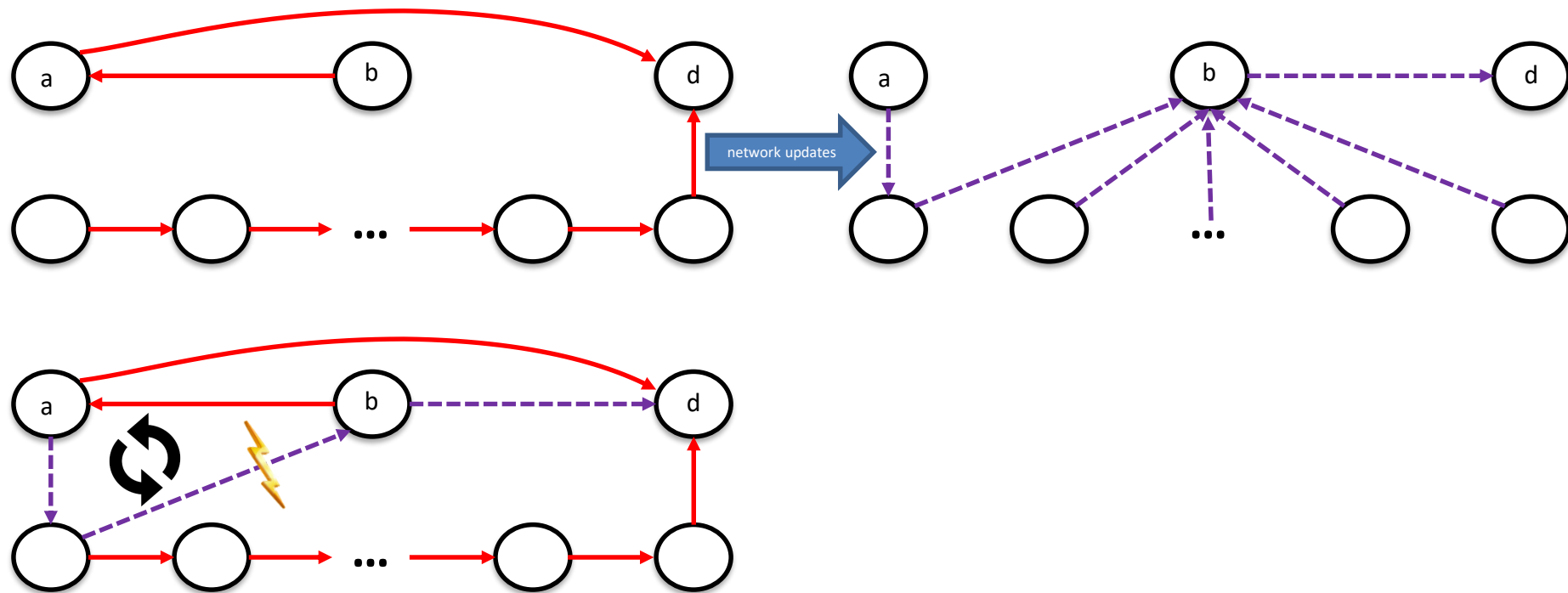


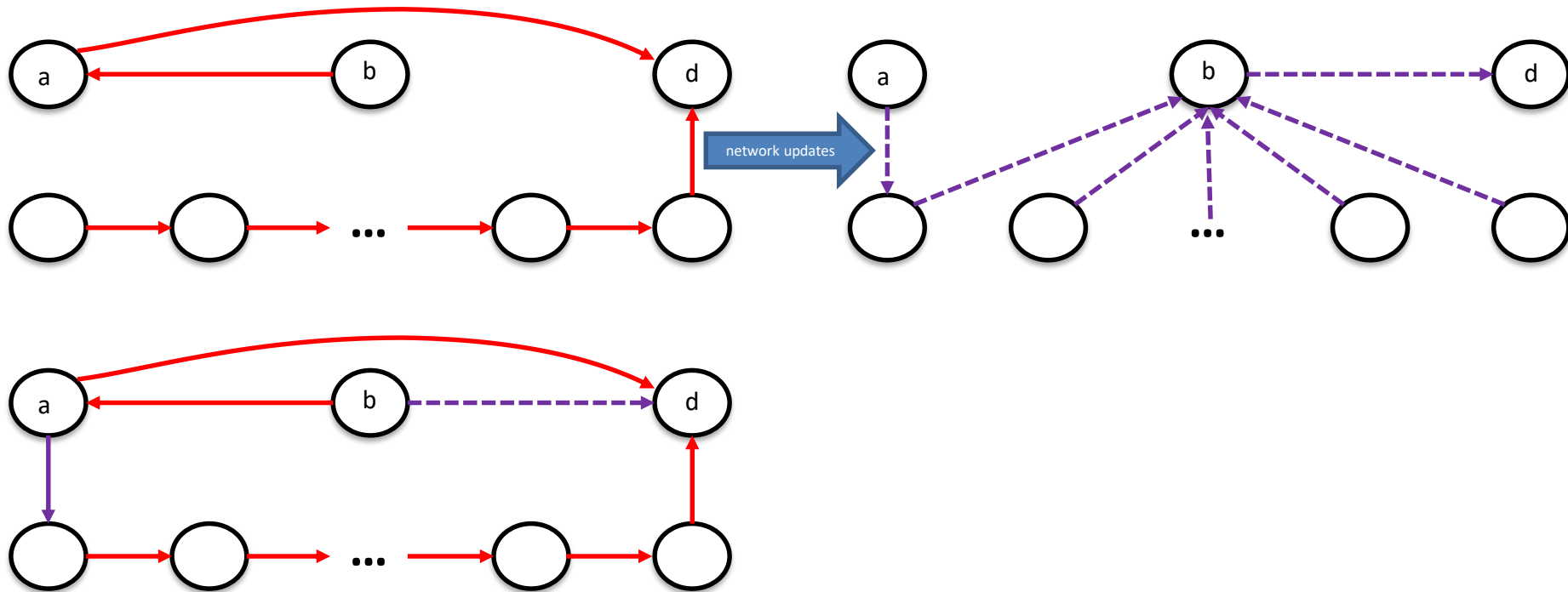




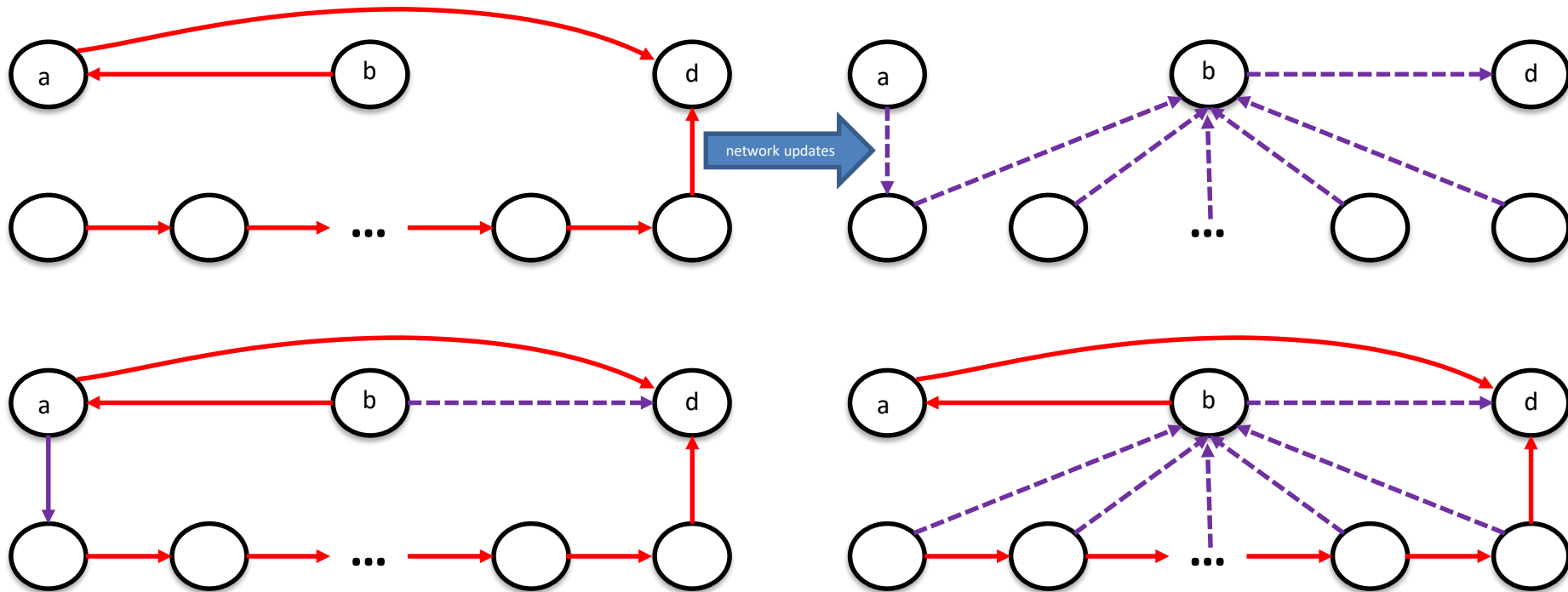








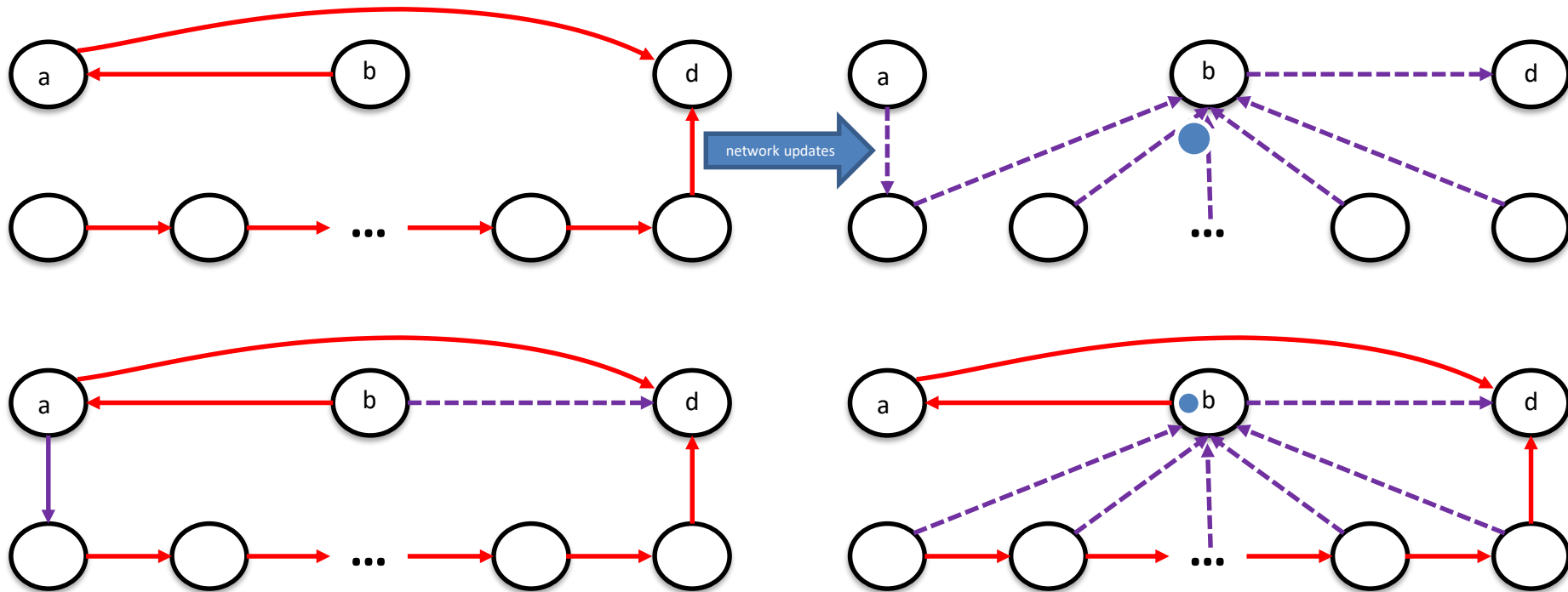
greedy **maximal** update  
 a & b update → all others wait  
**2** nodes update



greedy **maximal** update  
 a & b update → all others wait  
**2** nodes update

**maximum** update  
 a waits → all others update  
**all but 1** update

# How hard?

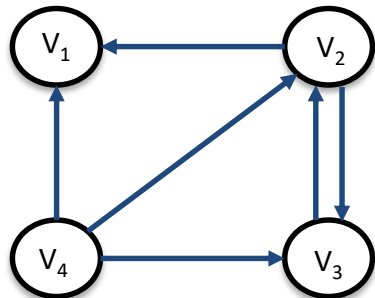


greedy **maximal** update  
 a & b update → all others wait  
**2 nodes** update

**maximum** update  
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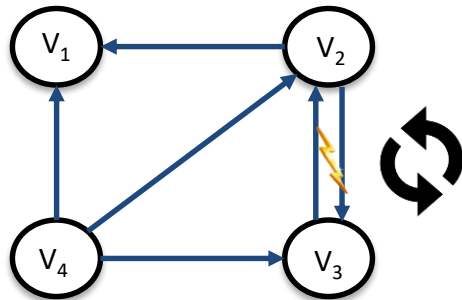
# Find maximum update?

- Let's go more general
- Delete all cycles in a graph



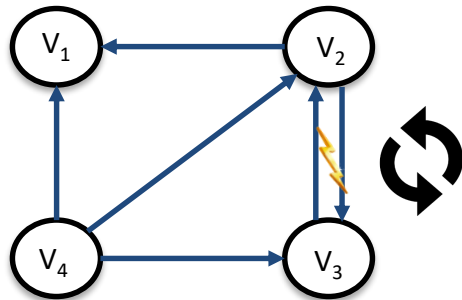
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# Find maximum update?

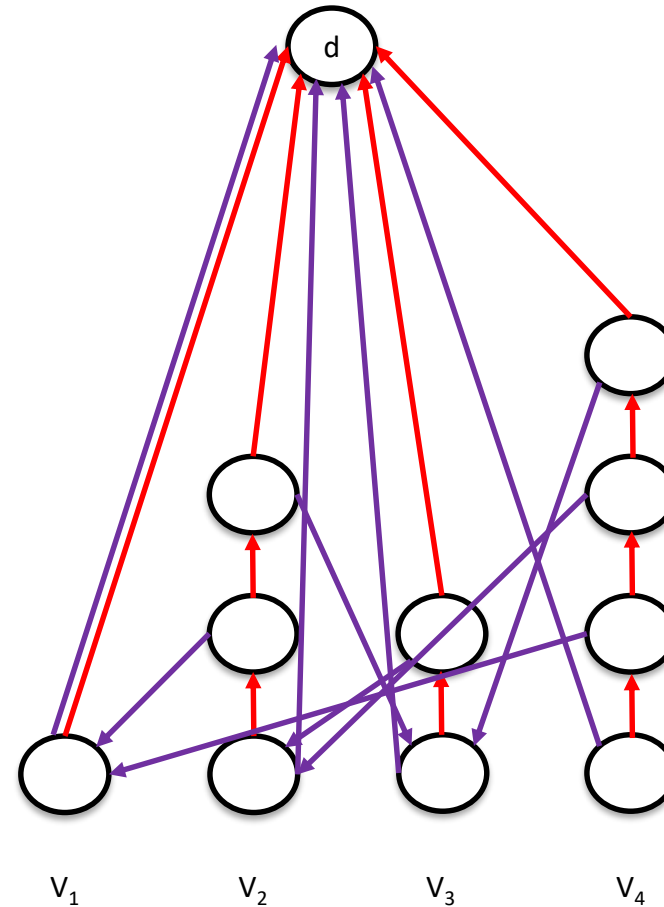
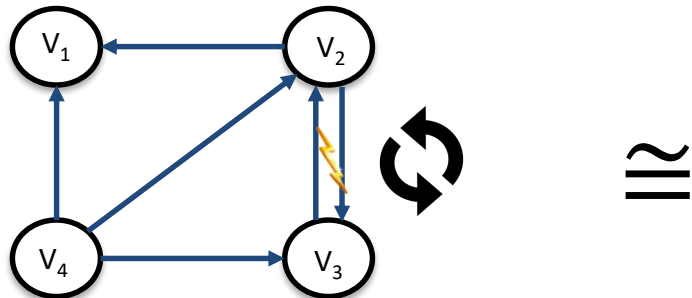
- Let's go more general
- Delete all cycles in a graph
- **NP-hard** to approximate
  - *Feedback Arc Set*





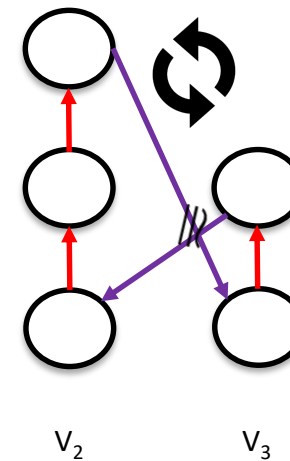
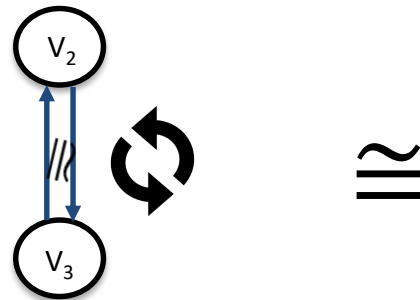
# Find maximum update?

- Let's go more general
- Delete all cycles in a graph
- **NP-hard** to approximate
  - *Feedback Arc Set*
- And it's (essentially) equivalent ☹️



# Find maximum update?

- Let's go more general
- Delete all cycles in a graph
- **NP-hard** to approximate
  - *Feedback Arc Set*
- And it's (essentially) equivalent ☹️



## Greedy? Update as many as possible per round

- Always works 😊
- Maximizing is NP-hard 😞
  - *Transiently Consistent SDN Updates: Being Greedy is Hard*. S. Akhoondian Amiri, A. Ludwig, J. Marcinkowski, S. Schmid. In: SIROCCO 2016
  - *The Power of Two in Consistent Network Updates: Hard Loop Freedom, Easy Flow Migration*. K.-T. Foerster, R. Wattenhofer. In: ICCCN 2016
- Single greedy update:  $O(1)$  rounds  $\Rightarrow \Omega(n)$  rounds 😞 😞
  - *Loop-Free Route Updates for Software-Defined Networks*. K.-T. Foerster, A. Ludwig, J. Marcinkowski, S. Schmid. In: IEEE/ACM Trans. Netw. 2018
- In general: Does a 3-round schedule exist? NP-hard 😞 😞 😞
  - *Loop-Free Route Updates for Software-Defined Networks*. K.-T. Foerster, A. Ludwig, J. Marcinkowski, S. Schmid. In: IEEE/ACM Trans. Netw. 2018



**Relax And Take it Easy!**



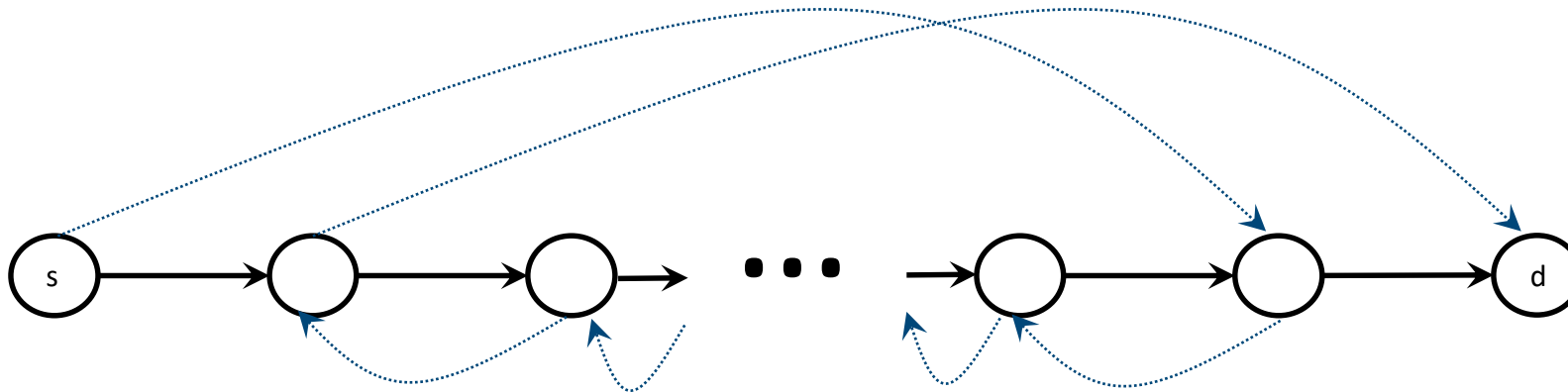


## Scheduling Loop-free Network Updates: It's Good to Relax! [Ludwig et al., PODC 2015]

Two key ideas:

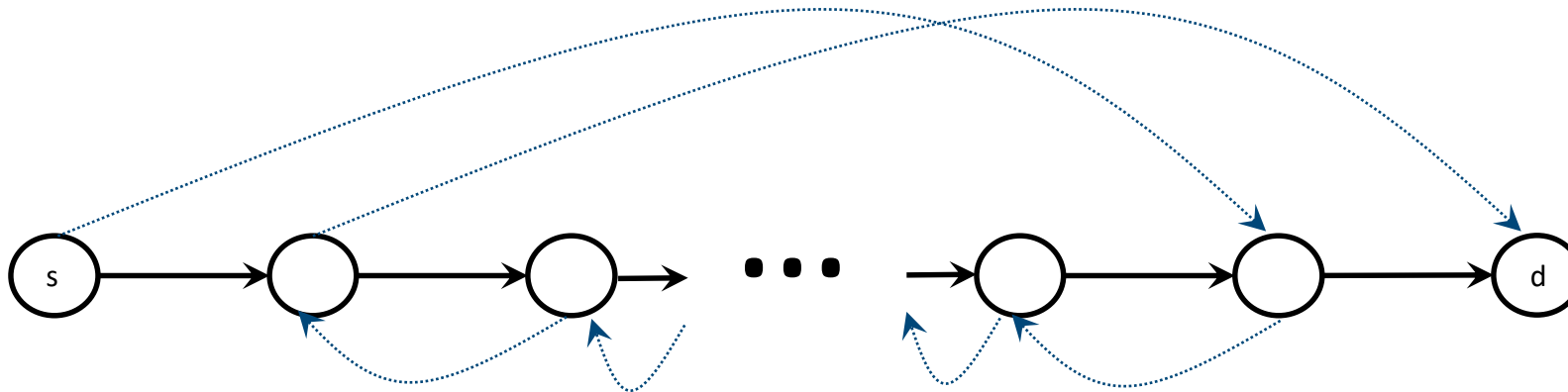
1. ~~destination  $d$  based~~ source-destination pairs  $\langle s, d \rangle$  . . .
2. ~~no forwarding loops~~ no loops between  $\langle s, d \rangle$

On its own:  
Makes 2-round updates  
polynomial, 3 still NP-hard



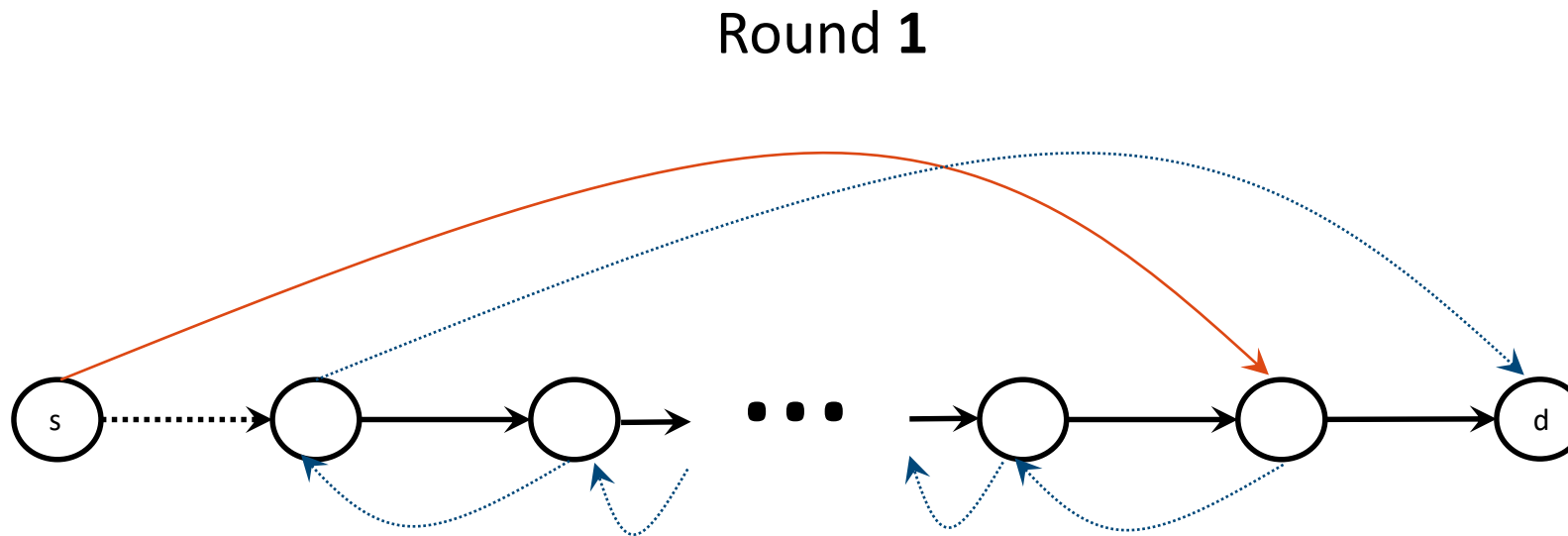
## Scheduling Loop-free Network Updates: It's Good to Relax!

- Non-relaxed?  $\Omega(n)$  rounds
- Relaxed?



## Scheduling Loop-free Network Updates: It's Good to Relax!

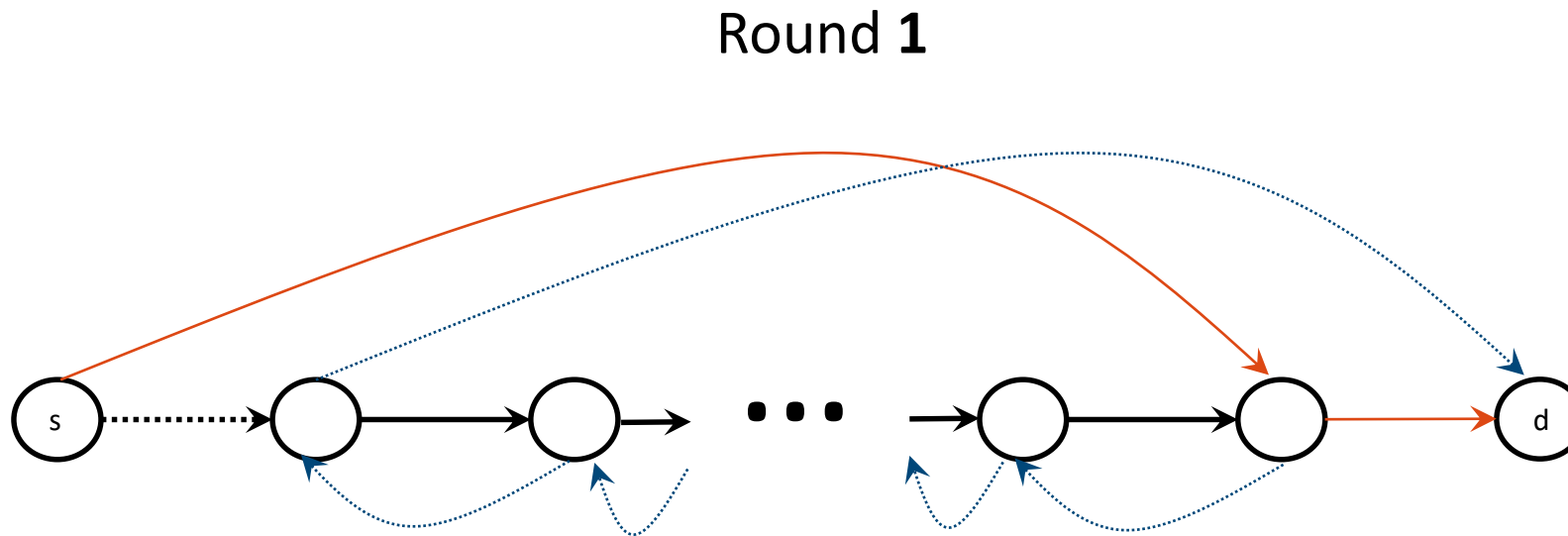
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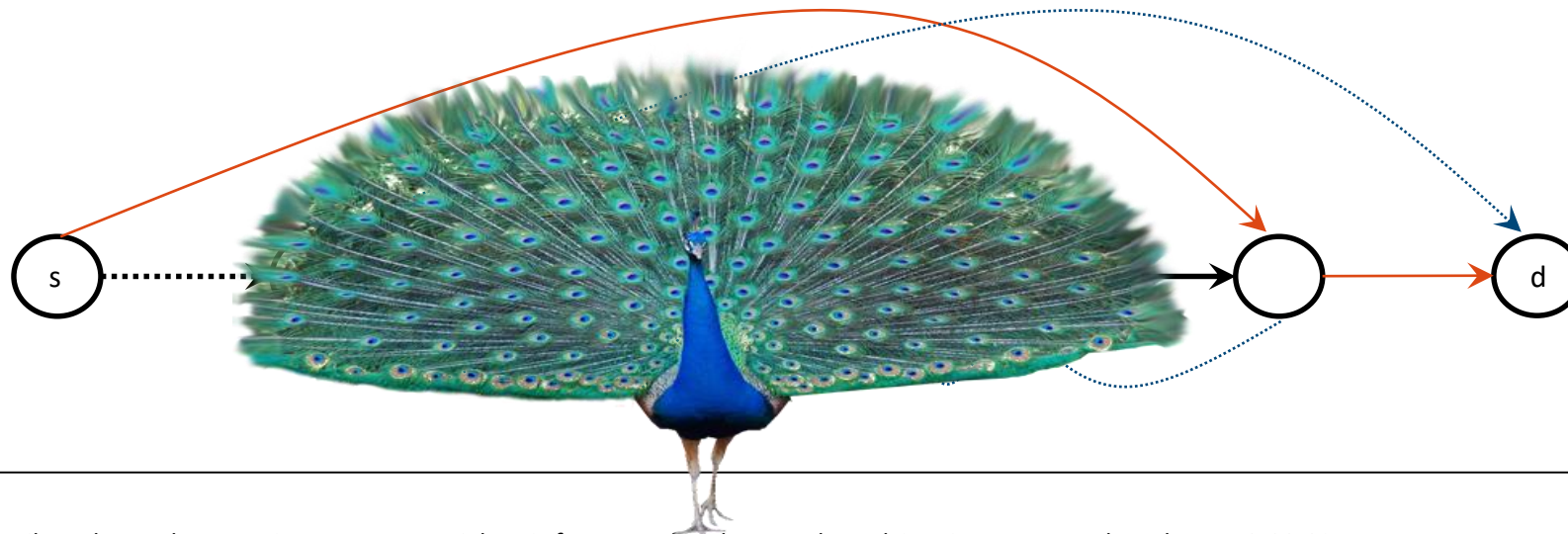
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## Scheduling Loop-free Network Updates: It's Good to Relax!

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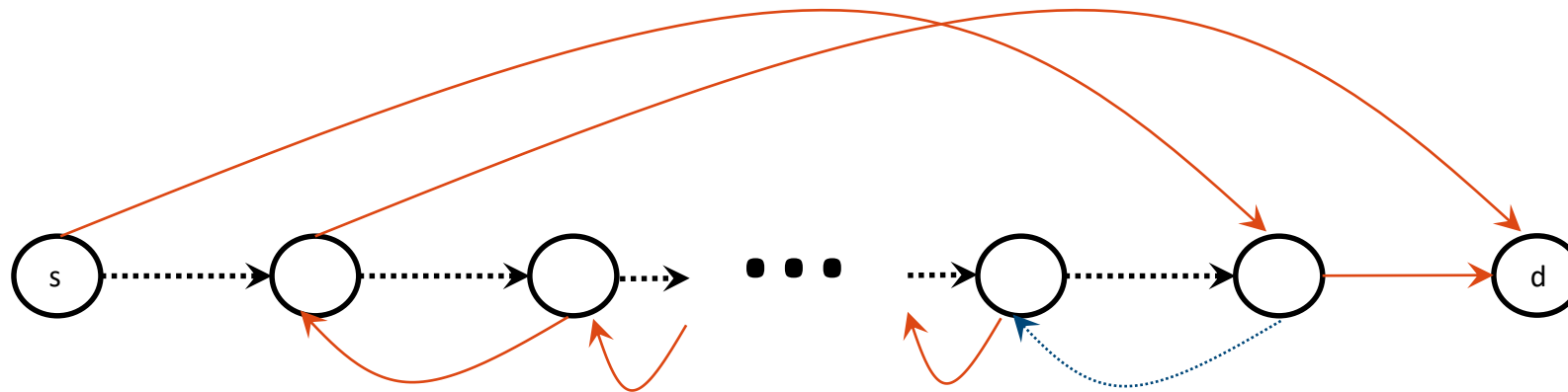
Round 1



## Scheduling Loop-free Network Updates: It's Good to Relax!

- Non-relaxed?  $\Omega(n)$  rounds
- Relaxed?

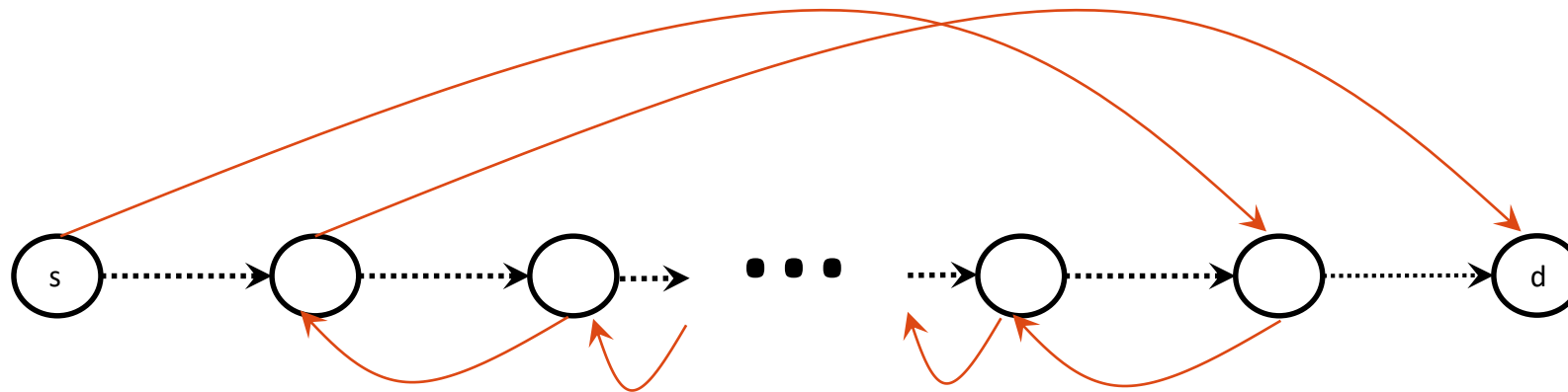
Round 2



## Scheduling Loop-free Network Updates: It's Good to Relax!

- Non-relaxed?  $\Omega(n)$  rounds
- Relaxed? Just 3 rounds

Round 3

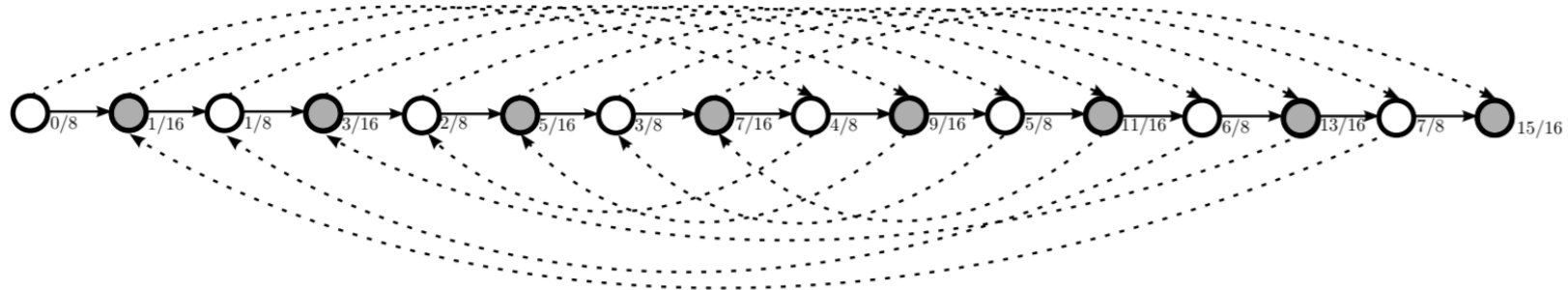


## Scheduling Loop-free Network Updates: It's Good to Relax!

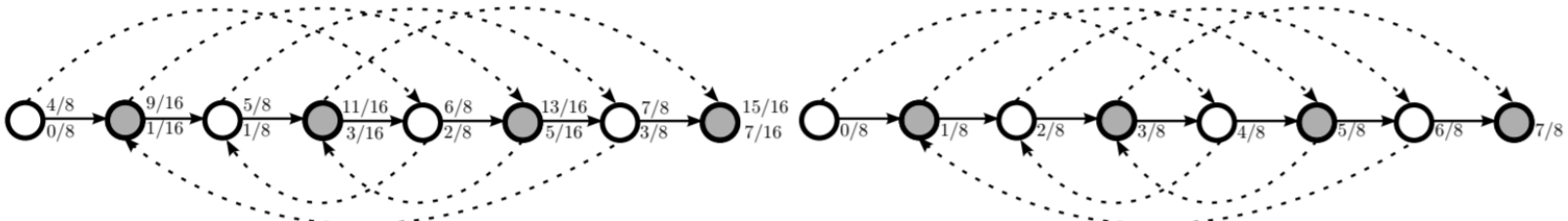
- Non-relaxed?  $\Omega(n)$  rounds
- Relaxed? Just 3 rounds
  - In general:  $O(\log n)$  rounds (“Peacock”)

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*Loop-Free Route Updates for Software-Defined Networks.* K.-T. Foerster, A. Ludwig, J. Marcinkowski, S. Schmid. In: IEEE/ACM Trans. Netw. 2018

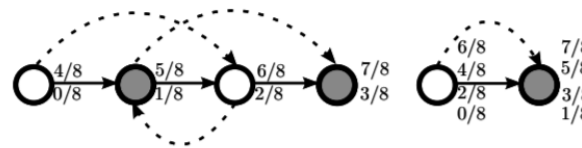


(a) The graph  $G_1$  with 16 nodes. When *Peacock* selects the edge from  $0/8$  to  $4/8$  as a shortcut, pruning results in the graph in Fig. 10b.



(b) After two rounds with *Peacock*, isomorphic to  $G_0$  in Fig. 10c.

(c) The graph  $G_0$  with 8 nodes.  $0/8$  to  $4/8$  is the next shortcut.



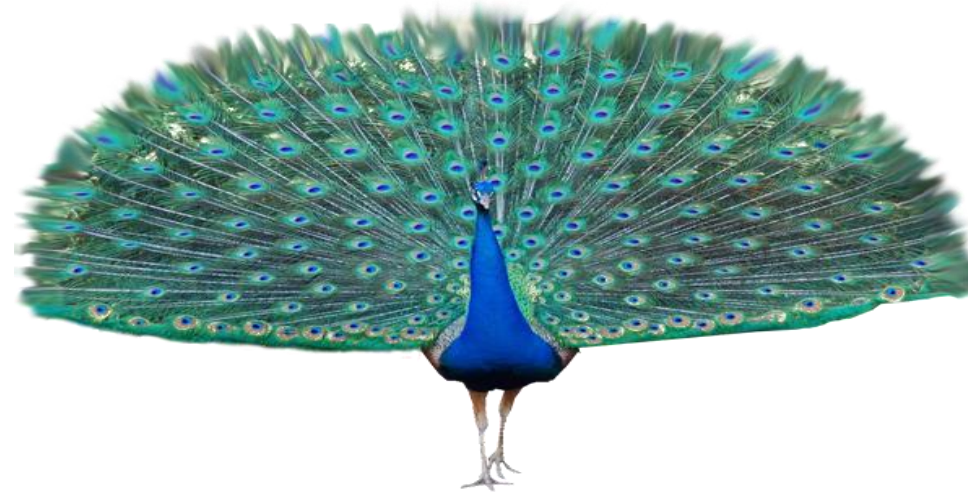
(d) To the left, the output of *Peacock* on  $G_0$  after two rounds. To the right, after two more rounds, selecting the first forward edge as a shortcut each time.



(e) The resulting updated graph, expanded into 16 nodes again.

## Scheduling Loop-free Network Updates: It's Good to Relax!

- Non-relaxed?  $\Omega(n)$  rounds
- Relaxed? Just 3 rounds
  - In general:  $O(\log n)$  rounds (“Peacock”)
  - But: Peacock instances with  $\Omega(\log n)$  rounds



*Loop-Free Route Updates for Software-Defined Networks.* K.-T. Foerster, A. Ludwig, J. Marcinkowski, S. Schmid. In: IEEE/ACM Trans. Netw. 2018

## Some Open Questions for scheduling loop free updates:

- For both models: Approximation algorithms for #rounds?

In a bit..

Relaxed:

- Optimal #rounds: NP-hard or in P?
- What is the real lower bound?

More open questions and specifics:  
*Survey of Consistent Software-Defined Network Updates*  
Klaus-Tycho Foerster, Stefan Schmid, Stefano Vissicchio  
*IEEE Communications Surveys & Tutorials*, 21(2), 2019

Non-relaxed:

- NP-hard for  $O(1) < k < \Omega(n)$  rounds?

Eg Congestion?  
Network functions?





## So Far Everything Was Sort of Centralized...

- ...can we make it more distributed?

## Decentralized Updates for „Tree-Ordering“

- So far: every round:
  - Controller computes and sends out updates
  - Switches implement them and send acks
  - Controller receives acks

## Decentralized Updates for „Tree-Ordering“

- So far: every round:
  - Controller computes and sends out updates
  - Switches implement them and send acks
  - Controller receives acks
  
- Alternative: Use dualism to so-called *proof labeling schemes*

Centralized Controller  
(Prover)



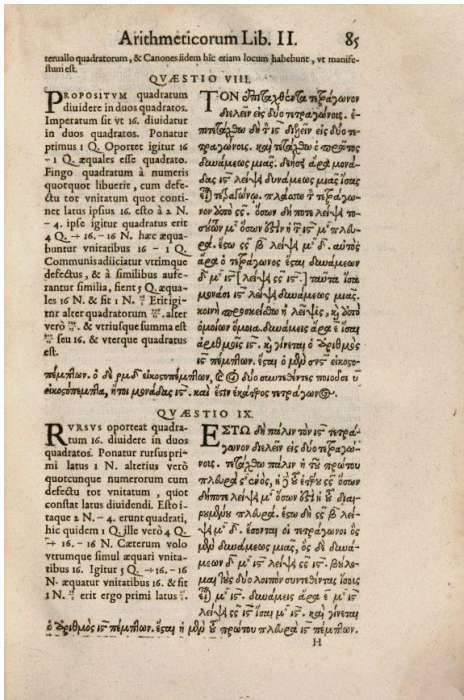
Eg P4 switch  
(Verifier)



# INTERMISSION

Proof-Labeling Schemes

# Deciding vs Checking



Prove



Verify



Annals of Mathematics, 142 (1995), 443-551

## Modular elliptic curves and Fermat's Last Theorem

By ANDREW WILES\*

For Nada, Clare, Kate and Olivia

*Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.*

Pierre de Fermat

### Introduction

An elliptic curve over  $\mathbb{Q}$  is said to be modular if it has a finite covering by a modular curve of the form  $X_0(N)$ . Any such elliptic curve has the property that its Hasse-Weil zeta function has an analytic continuation and satisfies a functional equation of the standard type. If an elliptic curve over  $\mathbb{Q}$  with a given  $j$ -invariant is modular then it is easy to see that all elliptic curves with the same  $j$ -invariant are modular (in which case we say that the  $j$ -invariant is modular). A well-known conjecture which grew out of the work of Shimura and Taniyama in the 1950's and 1960's asserts that every elliptic curve over  $\mathbb{Q}$  is modular. However, it only became widely known through its publication in a paper of Weil in 1967 [We] (as an exercise for the interested reader!), in which, moreover, Weil gave conceptual evidence for the conjecture. Although it had been numerically verified in many cases, prior to the results described in this paper it had only been known that finitely many  $j$ -invariants were modular.

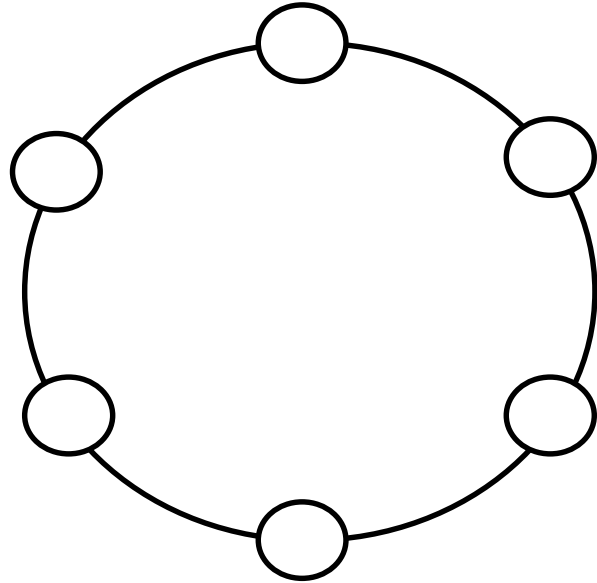
In 1985 Frey made the remarkable observation that this conjecture should imply Fermat's Last Theorem. The precise mechanism relating the two was formulated by Serre as the  $\epsilon$ -conjecture and this was then proved by Ribet in the summer of 1986. Ribet's result only requires one to prove the conjecture for semistable elliptic curves in order to deduce Fermat's Last Theorem.

\*The work on this paper was supported by an NSF grant.

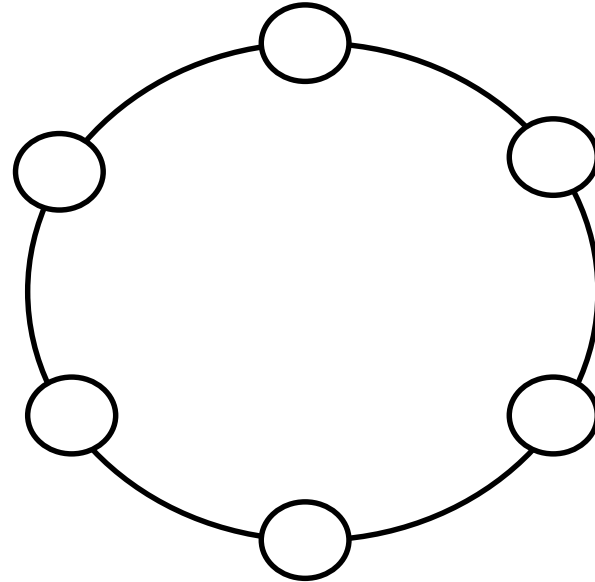
## Brief Selected Background

- [Naor and Stockmeyer, STOC 1993]:  
*What can be computed locally?*
- [Korman et al., PODC 2005]:  
*Proof Labeling Schemes (PLS)*
- [Göös and Suomela, PODC 2011]:  
*Locally Checkable Proofs (LCP)*
- [Fraigniaud et al., FOCS 2011,...]:  
*Nondeterministic Local Decision (NLD)*
- And many more recent works, e.g., on approximation, randomization etc.


## Example



## Example

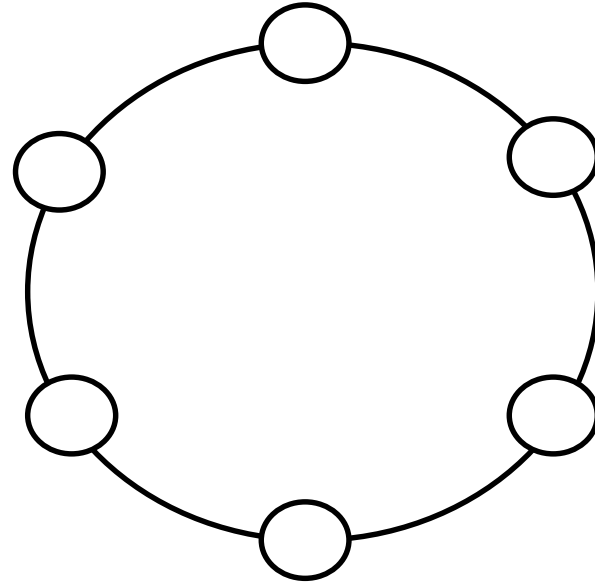


### Model

- Each of the  $n$  nodes  is a computer, connected by links
- Synchronous rounds
  - Simplified: unlimited message size & computational power, unique identifiers for nodes

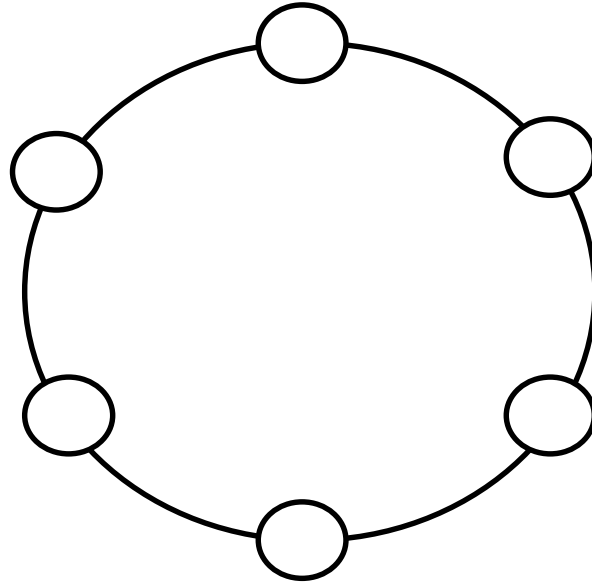


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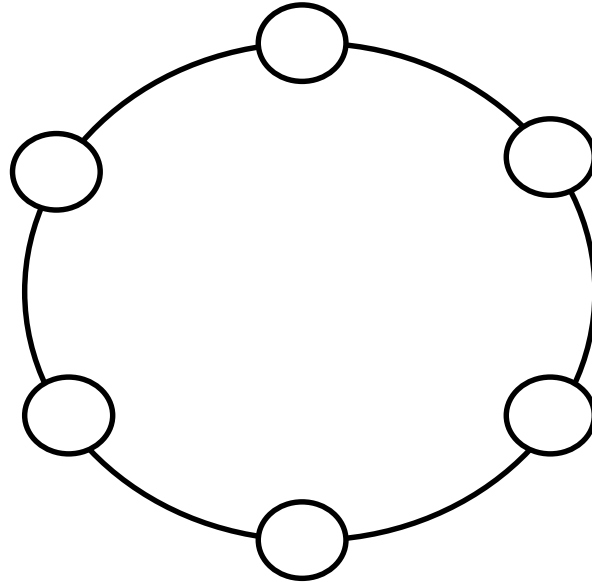
- Is  $n$  even?

## Example



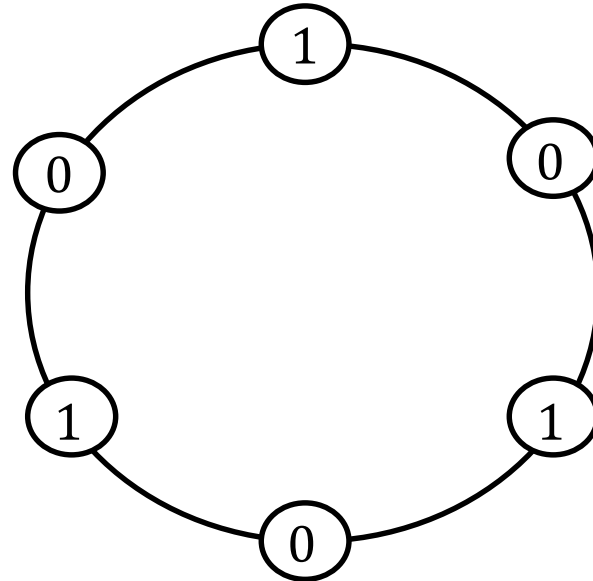
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- $\Omega(n)$  rounds

## Example



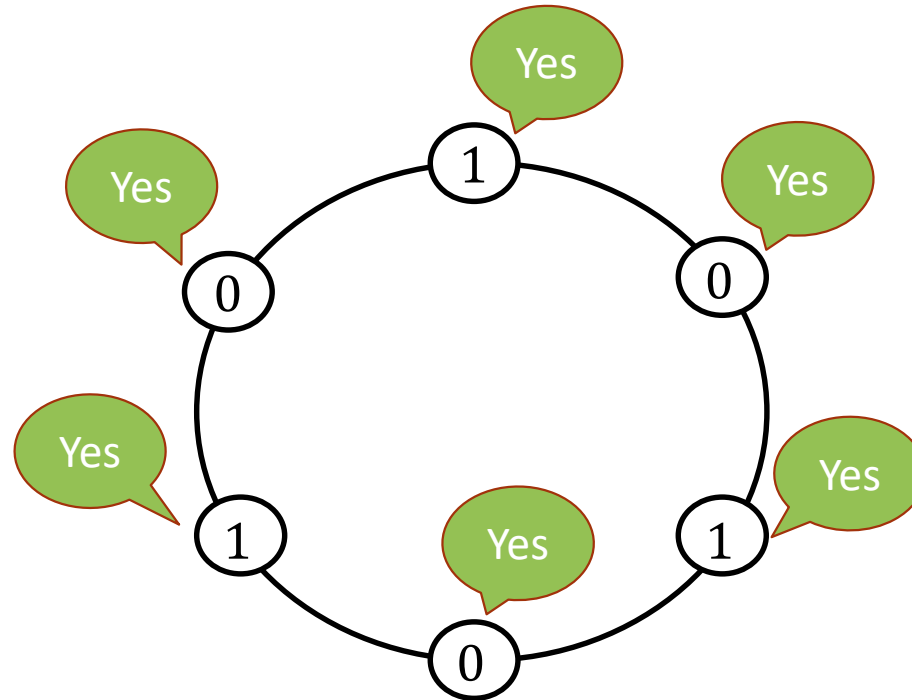
- Is  $n$  even?
- $\Omega(n)$  rounds
- What if I tell you it is even? Why should you trust me 😊

## Example



- Is  $n$  even?
- $\Omega(n)$  rounds
- $\mathcal{P}$ rover assigns 1 bit?

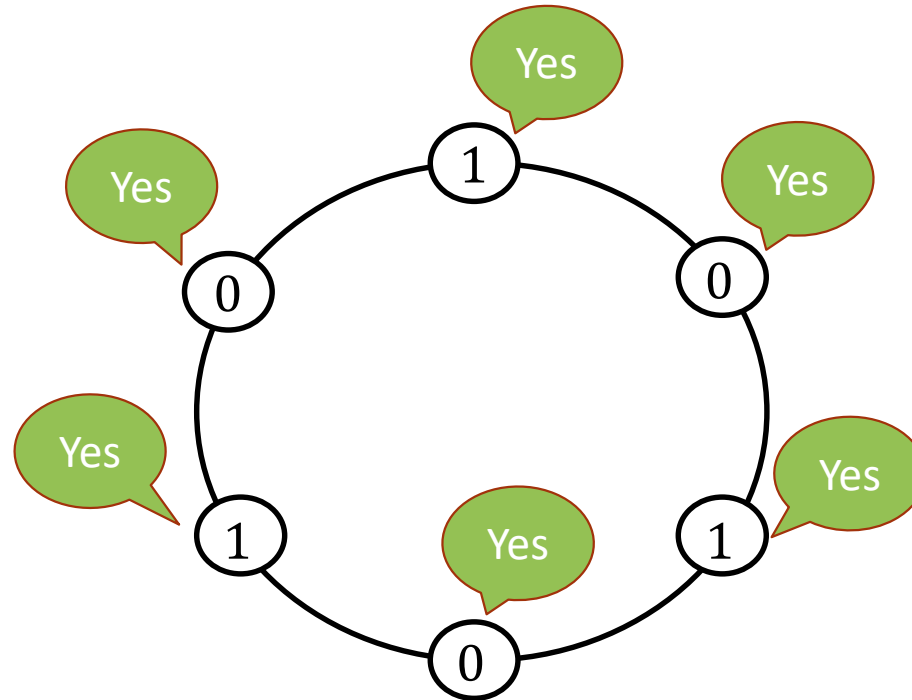
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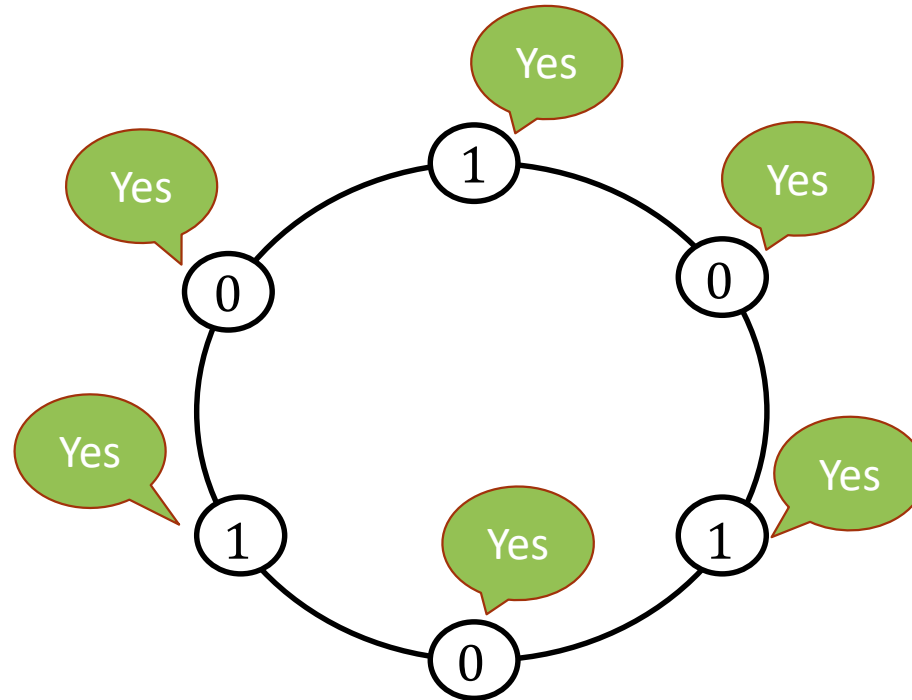
## Example



- Is  $n$  even?
- $\Omega(n)$  rounds
- Prover assigns 1 bit  $\rightarrow$  Verify in 1 round
- Other way to think of it: 1 bit of non-determinism
- General question: How many bits necessary/sufficient?

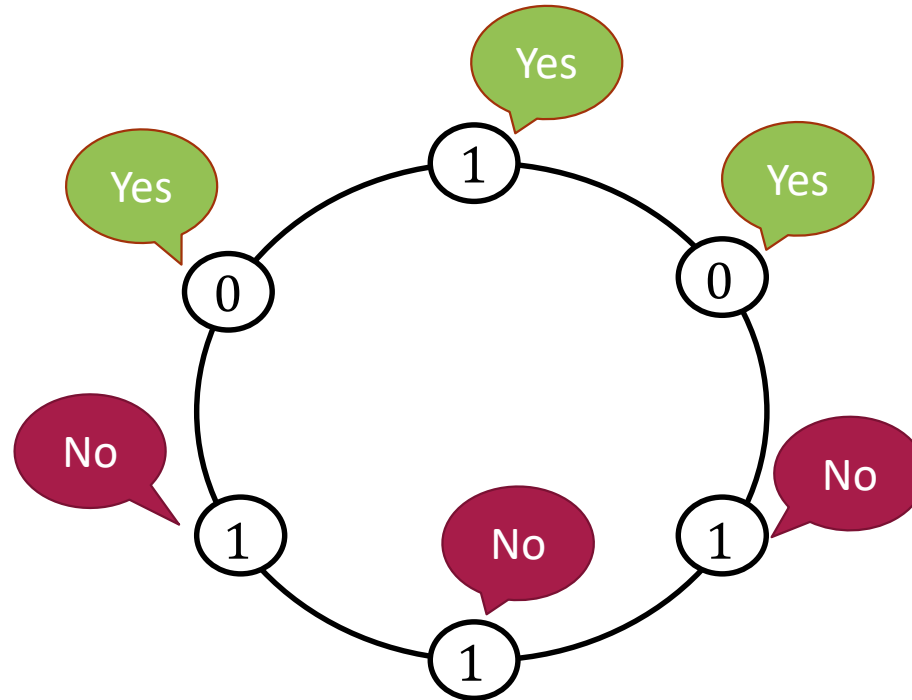


## Accepting a proof



- Every node outputs **Yes** -> Proof accepted
- One node outputs **No** -> Proof rejected

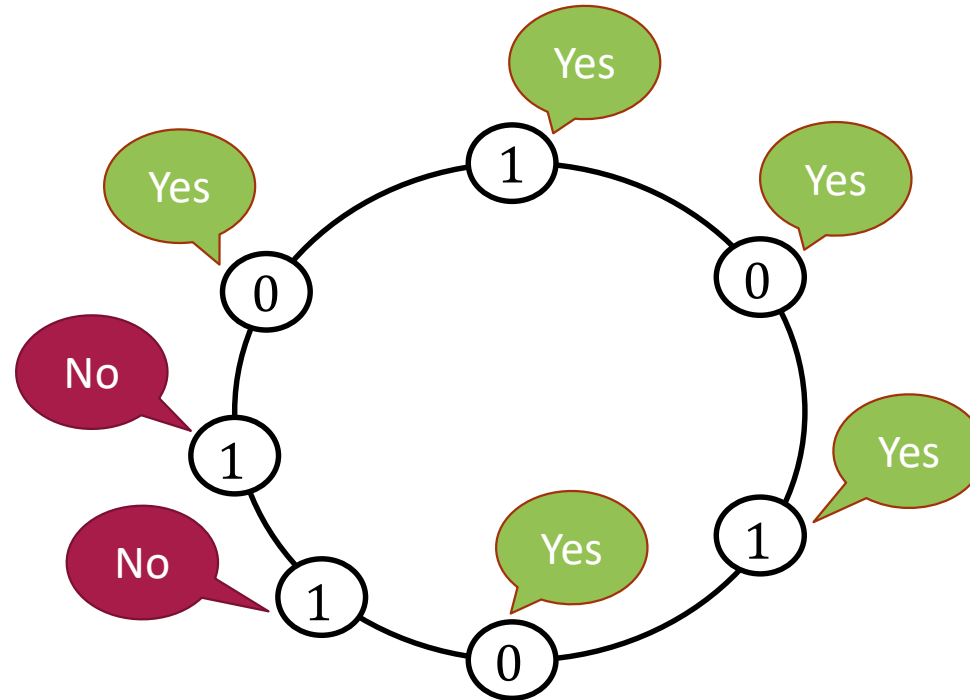
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  - *Prover* chose the wrong proof



## Accepting a proof



- Every node outputs **Yes** -> Proof accepted
- One node outputs **No** -> Proof rejected
  - Prover chose the wrong proof
  - Property does not hold

Back to SDNs: Switch from a proof to another

## Decentralized Updates for “Self-Organizing”

When should I update?



## Decentralized Updates for “Self-Organizing”

Once my parent updates!



## Decentralized Updates for “Parenting”

Once my parent updates!



Send parent ID



## Decentralized Updates for „Tree-Ordering“



## Decentralized Updates for “Learning”



## Decentralized Updates for “Tree-Ordering”

- + Only one controller-switch interaction per route change
- + New route changes can be pushed before old ones done (*include “version#”*)
- + Incorrect updates can be locally detected (*include depth in tree, prevents loops*)
- +/- Speed benefit/penalty depends on update scenario and topology
- Requires switch-to-switch communication e.g., [Nguyen et al., SOSR 2017]

K.-T. Foerster, T. Luedi, J. Seidel, R. Wattenhofer: *Local Checkability, No Strings Attached: (A)cyclicity, Reachability, Loop Free Updates in SDNs* . In: Theoret. Comput. Sci. 2018

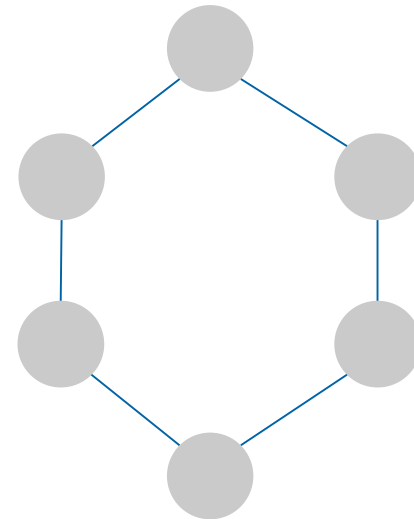
K.-T. Foerster, S. Schmid: *Distributed Consistent Network Updates in SDNs: Local Verification for Global Guarantees*. Under submission.

## Can we also make the initial computation decentralized?

- Classic setting of distributed computing (e.g. LOCAL or CONGEST model)
  - Possible benefit in SDNs:
    - We do not need to compute from scratch!
      - In wired networks, problems depend on a subset of the network
        - Leverage **Preprocessing**
- Further explored in eg:
  - Exploiting Locality in Distributed SDN Control. S. Schmid, J. Suomela, HotSDN 2013
  - On the Power of Preprocessing in Decentralized Network Optimization. K.-T. Foerster, J. Hirvonen, S. Schmid, J. Suomela, INFOCOM 2019
  - BA: Does Preprocessing help under Congestion? K.-T. Foerster, J. Korhonen, J. Rybicki, S. Schmid, PODC 2019

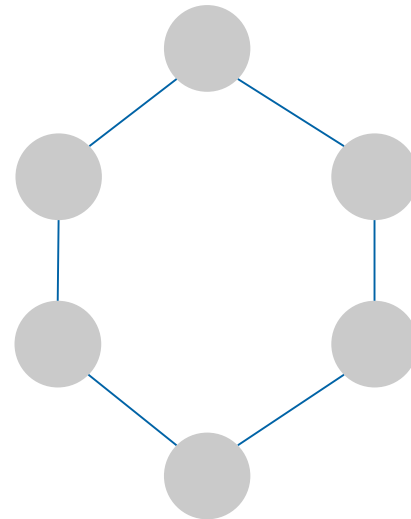


## Coloring of rings (LOCAL model)



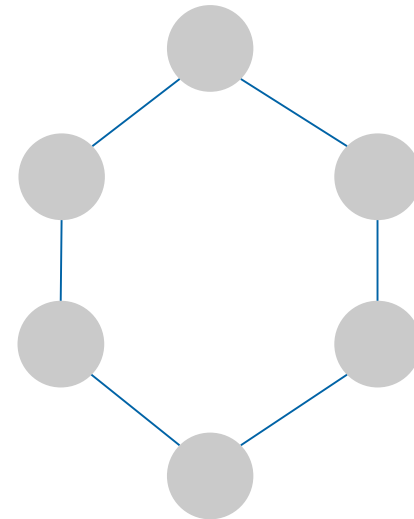
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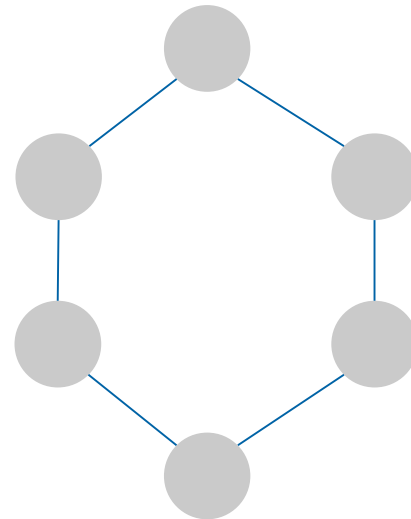
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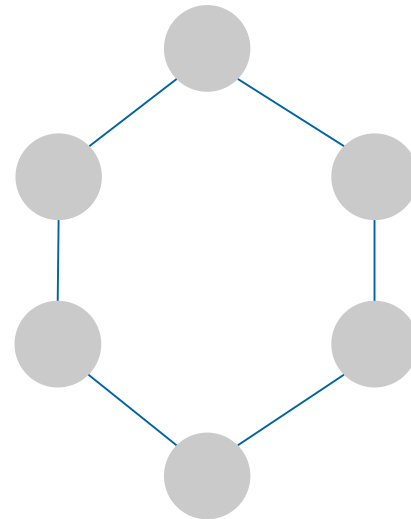
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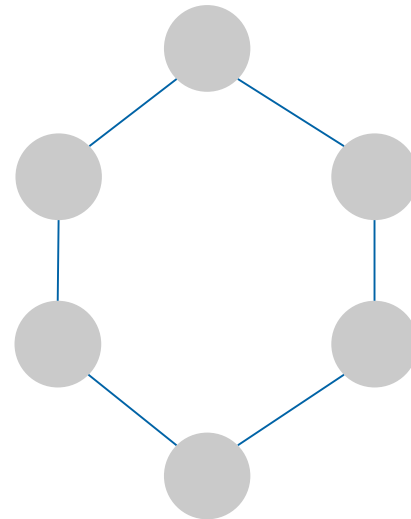
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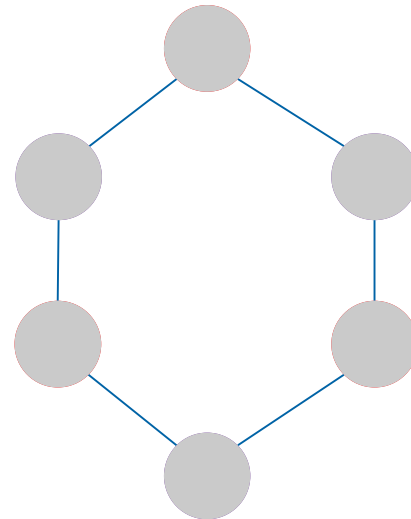
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- 2-coloring:
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- 3-coloring:
  - Needs non-constant time
- Cannot improve in the LOCAL model ☹️



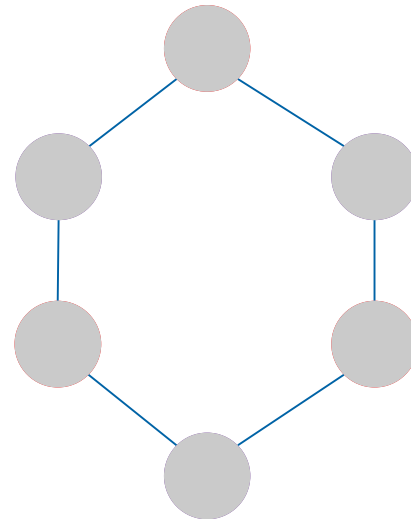
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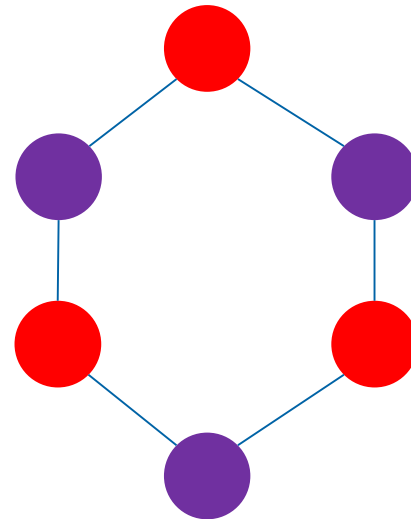
- 2-coloring:
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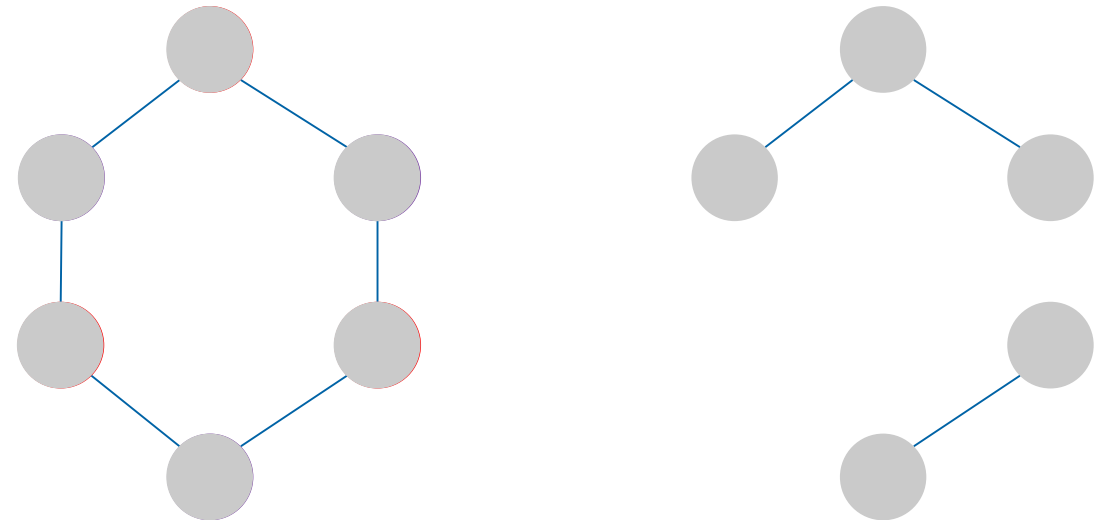
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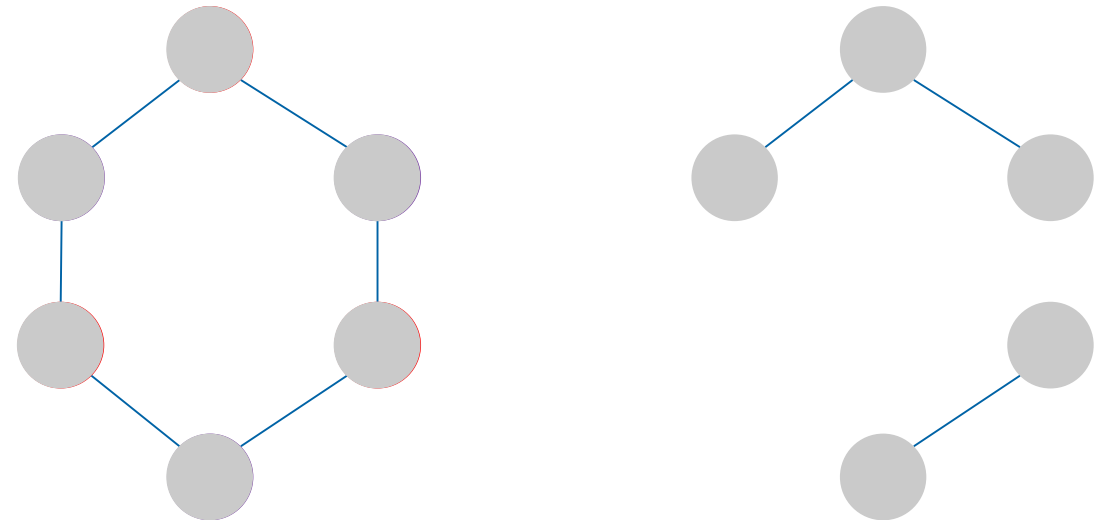
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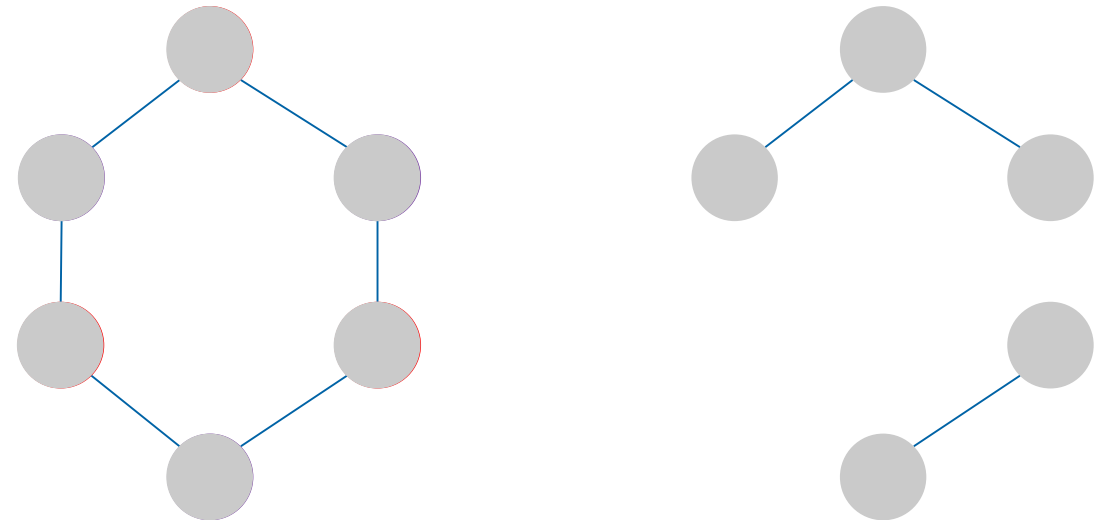
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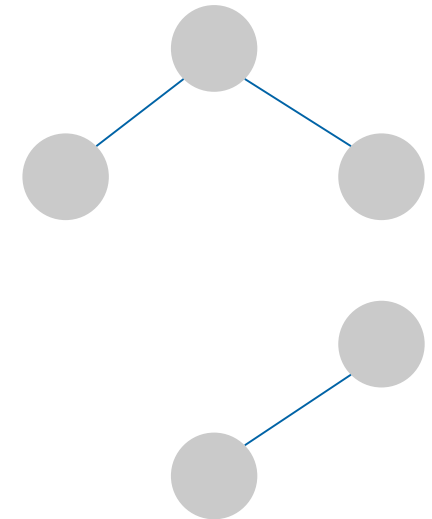
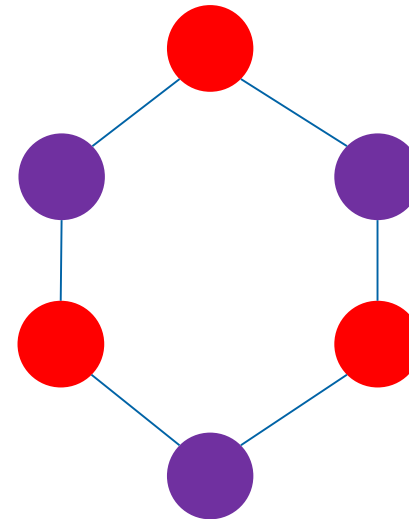
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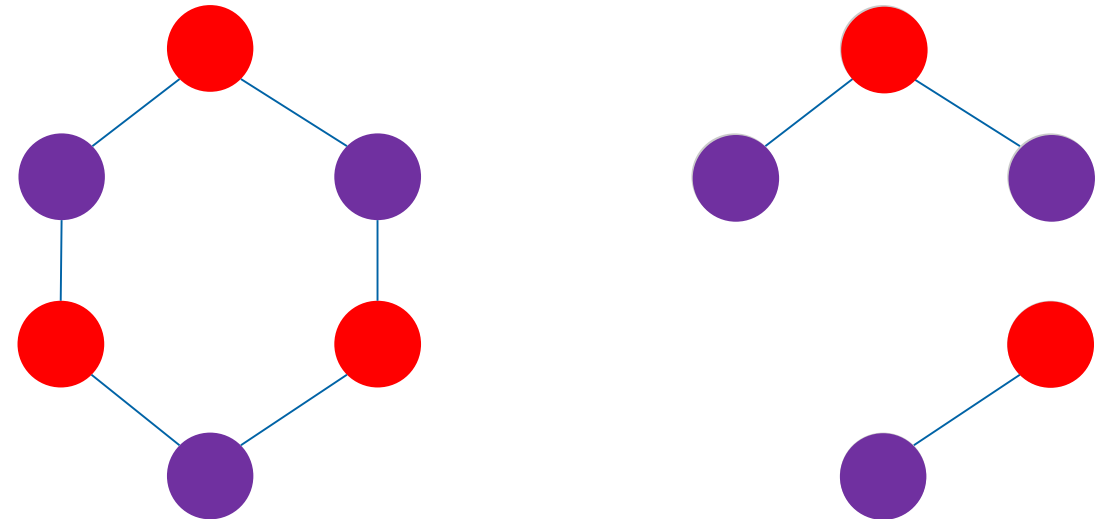
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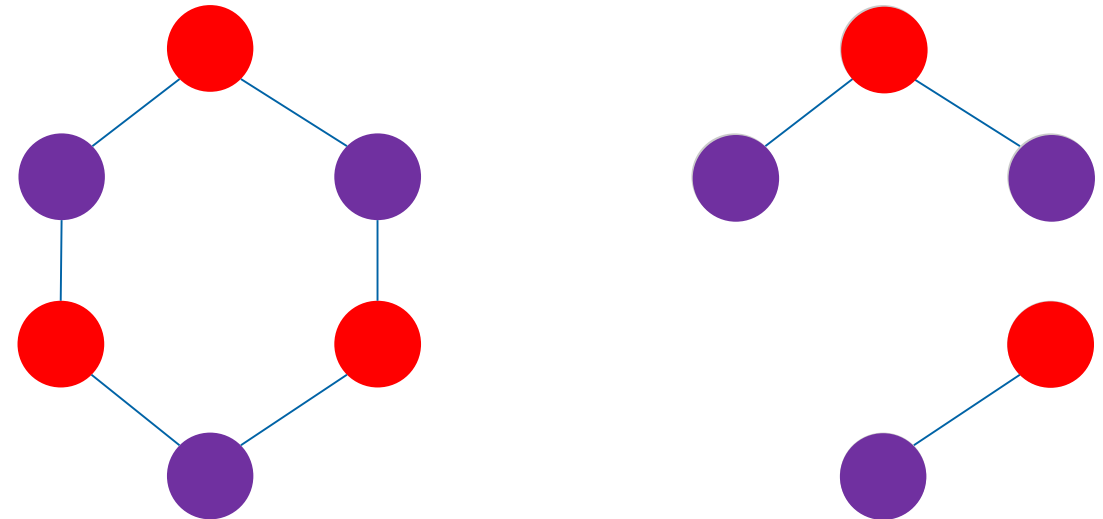
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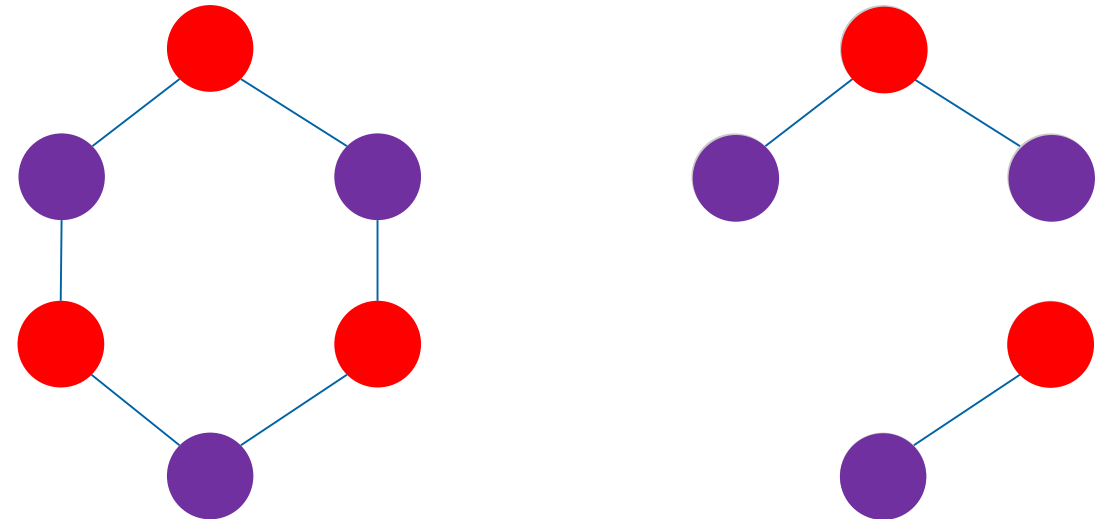
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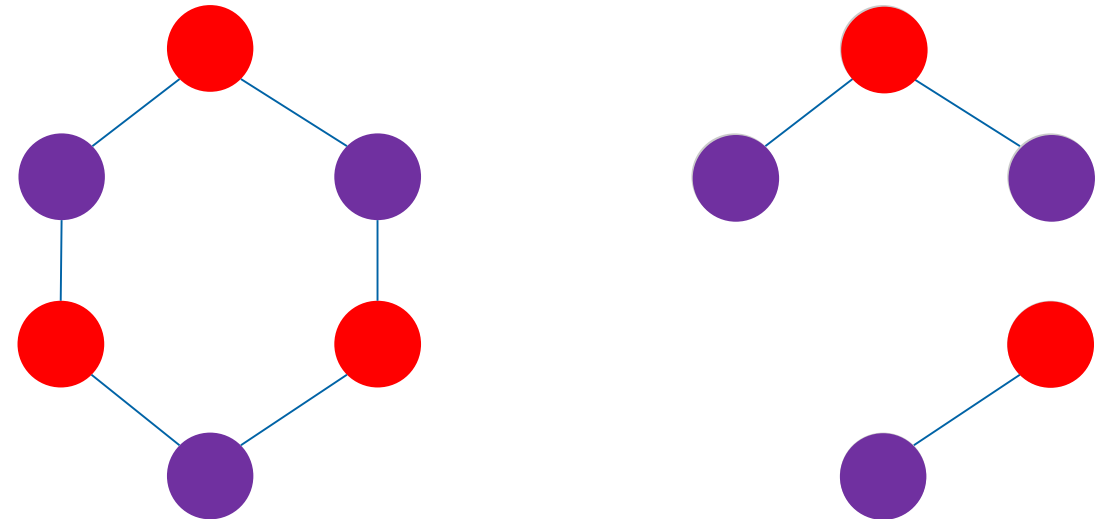
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- With preprocessing: fast!
  - Coloring remains valid
- What are further application scenarios?
- What else can we do with the SUPPORT of Preprocessing?



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- Example: Software-Defined Networking, single (logically centralized) controller does not scale
  - Create many local controllers that can react quickly, that control small set of “dumb” nodes

## The SUPPORTED Model

- Extends the LOCAL model (w. unique IDs) with preprocessing

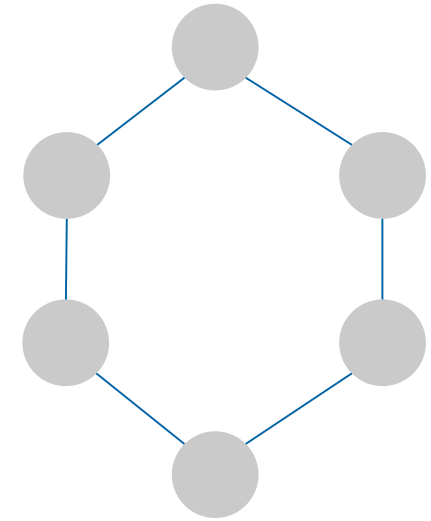
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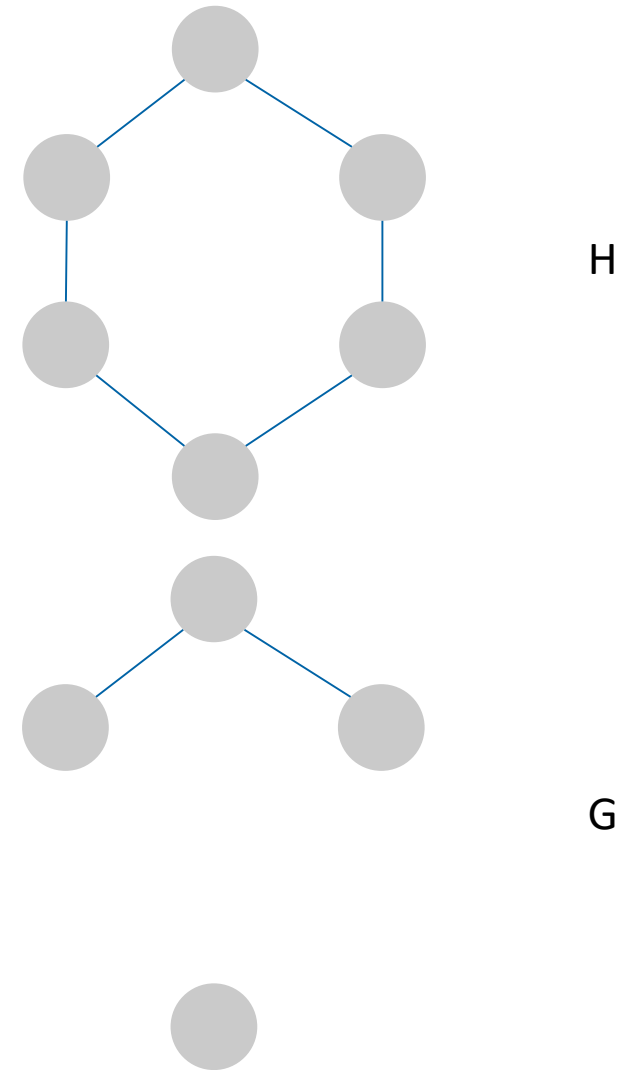
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H

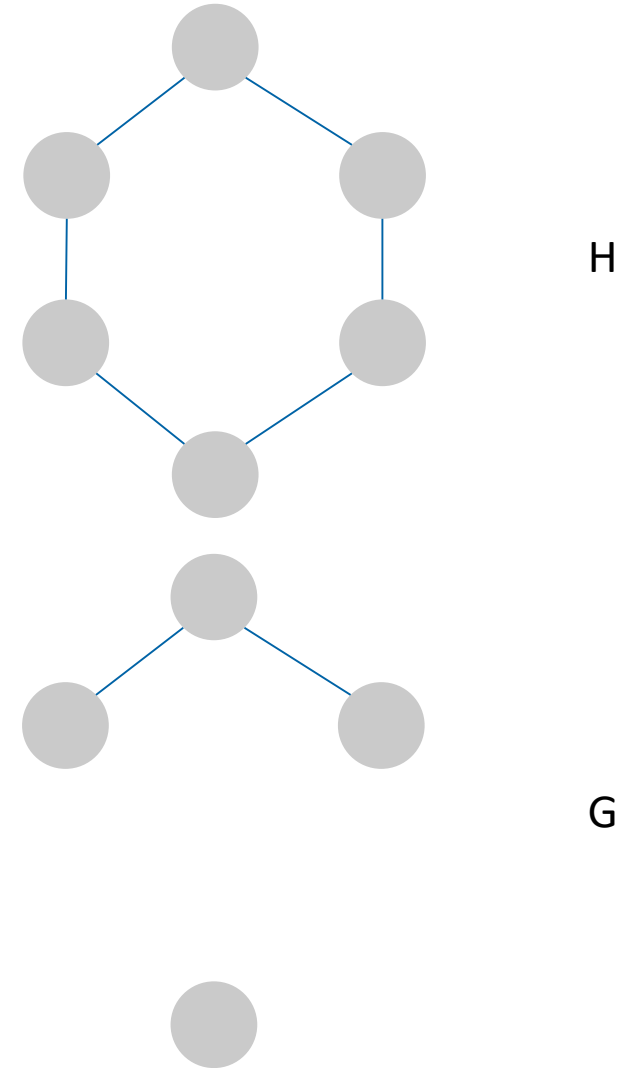
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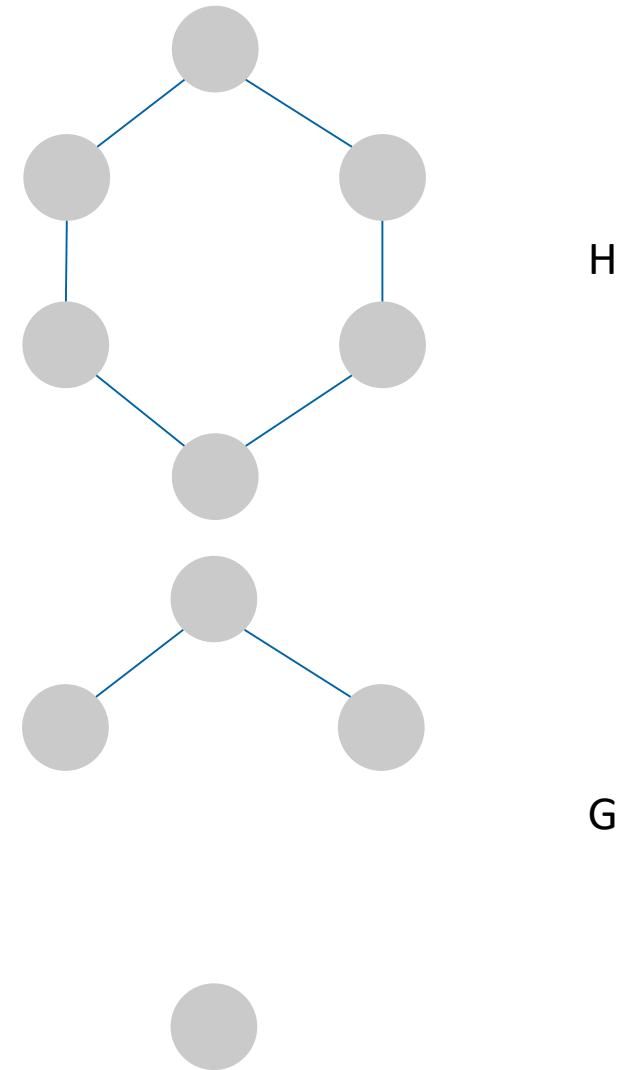
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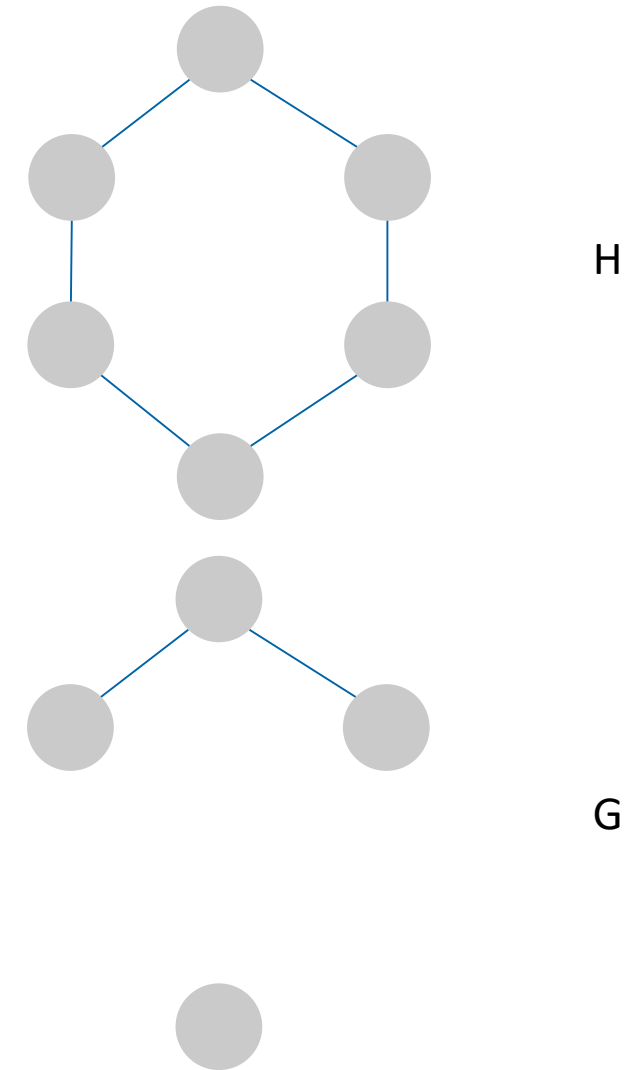
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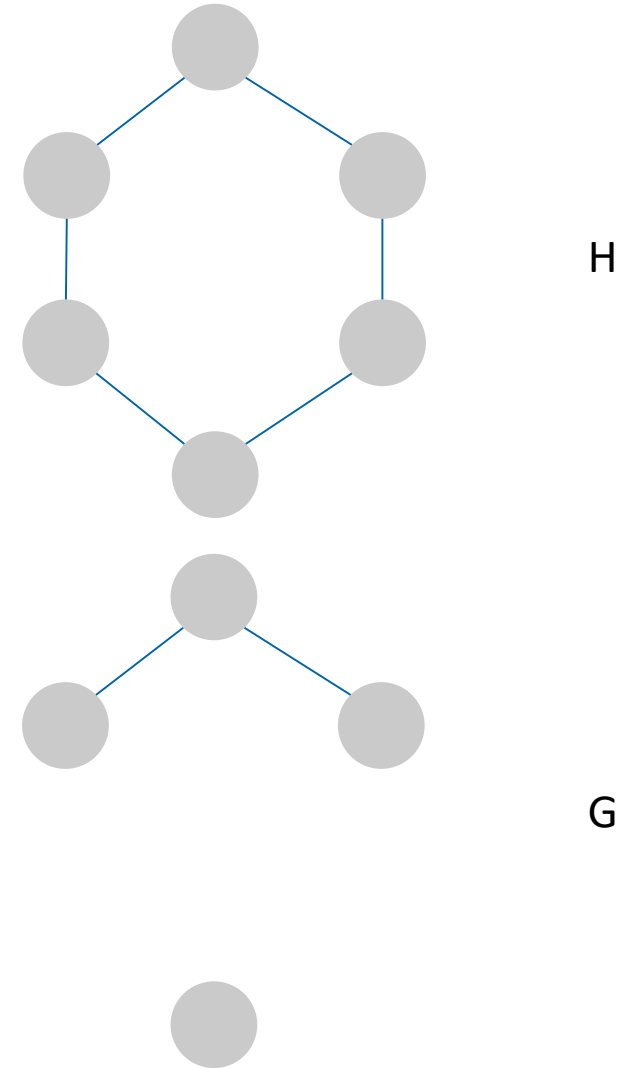
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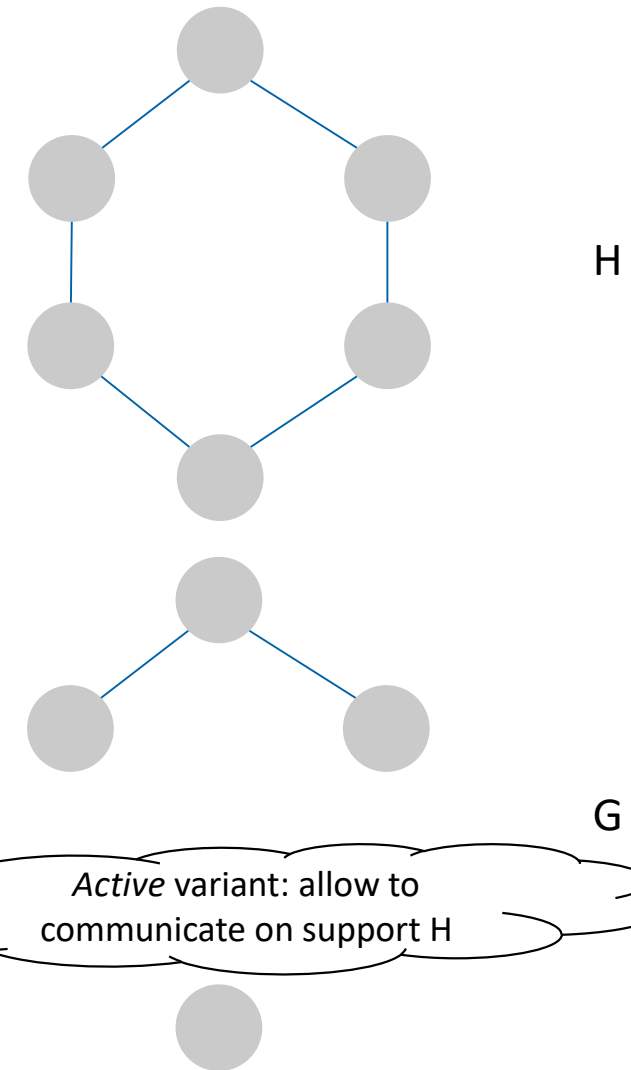


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## Does the SUPPORTED Model make everything easy?

- Task: Leader election ( $\Theta(\text{diameter})$  runtime in LOCAL model)
  - Easy if  $G=H$ : precompute leader, 0 rounds
  - But for different  $G$ :
    - We need to compute a leader for each connected component of  $G$ !
      - Component has no leader? Re-elect ☹️
      - Component has multiple leaders? Re-elect ☹️
      - Components can have asymptotically same diameter ☹️
- SUPPORTED model does not provide a “silver bullet”
  - Not even for the *active* variant



**Maybe even useless in general?**

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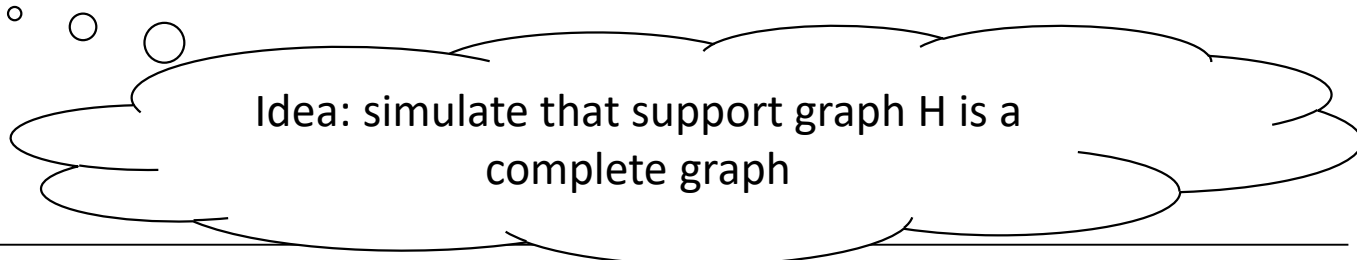
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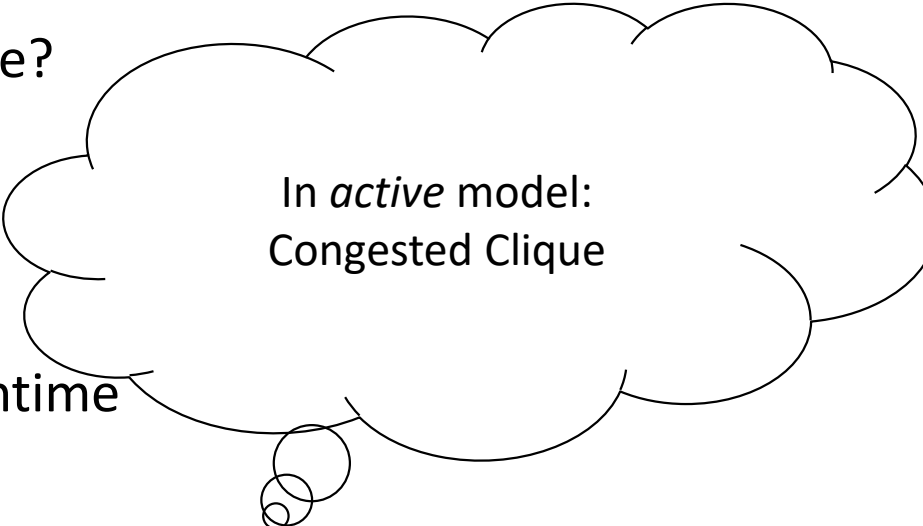
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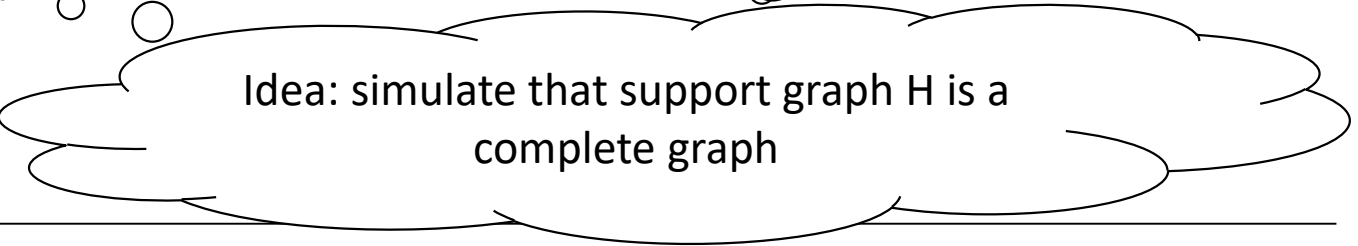
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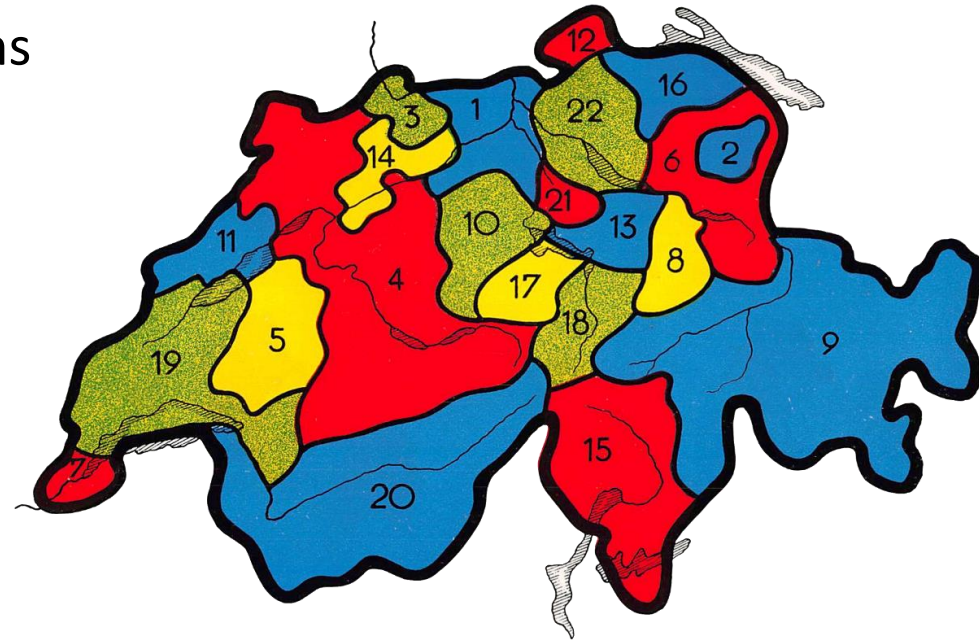
In *active* model:  
Congested Clique



Idea: simulate that support graph  $H$  is a  
complete graph

## But: Restricted Graph Families are Useful 😊

- Real topologies are usually not complete graphs
- Case study: planar graphs
  - Remain planar under edge deletions
  - Are 4-colorable



„Geloeste und ungeloeeste Mathematische Probleme aus alter und neuer Zeit" by Heinrich Tietze  
<http://www.math.harvard.edu/~knill/graphgeometry/faqg.html>

## Case Study: Dominating Set

- Task: Find subset  $D$  of nodes s.t. every node
  - Has a neighbor in  $D$  or is in  $D$
- Can we pre-compute?
  - A bad one yes: everyone in  $D$ !
  - But not an optimal one!
    - Graph can look very different



## Case Study: Minimum Dominating Set in Planar Graphs

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- $(1+\delta)$ -approximation not possible in constant time [Czygrinow et al., DISC 2008]



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    - 3-color pseudo-forest [non-constant time ☹️]
    - Run clustering/optimization algorithms on components of constant size [constant time 😊]
- SUPPORTED speed-up:  
1) precompute 4-coloring  
2) reduce 4-colored pseudo-forest to 3 colors in 2 rounds

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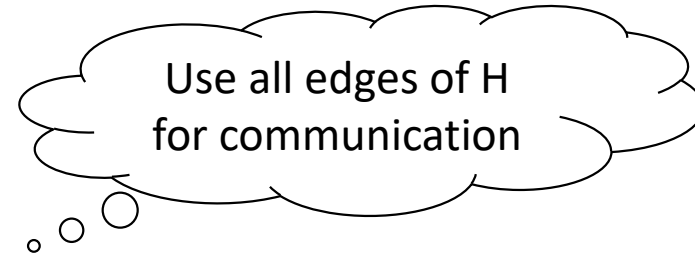
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  - Also for planar graphs for maximum independent set & maximum matching

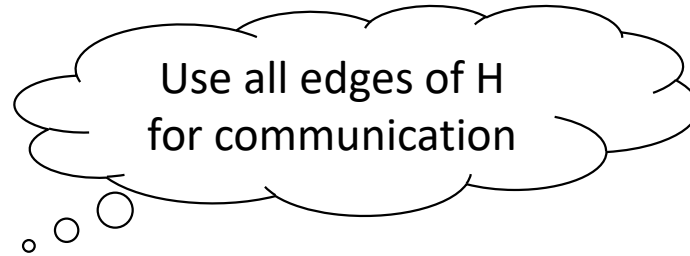


Max out-degree of 1

## Further Results in the *Active* SUPPORTED Model

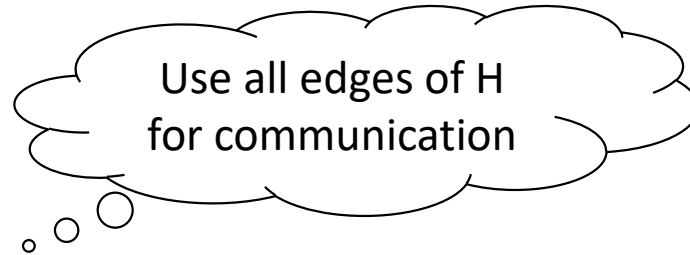


## Further Results in the *Active SUPPORTED* Model



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Use all edges of  $H$   
for communication

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937v1 [cs.DS] 25 Jul 2019

## Polylogarithmic-Time Deterministic Network Decomposition and Distributed Derandomization

Václav Rozhoň  
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rozho nv@student.ethz.ch

Mohsen Ghaffari\*  
ETH Zurich  
ghaffari@inf.ethz.ch

### Abstract

We present a simple polylogarithmic-time deterministic distributed algorithm for network decomposition. This improves on a celebrated  $2^{O(\sqrt{\log n})}$ -time algorithm of Panconesi and Srinivasan [STOC'93] and settles one of the long-standing and central questions in distributed graph algorithms. It also leads to the first polylogarithmic-time deterministic distributed algorithms for numerous other graph problems, hence resolving several open problems, including Linial's well-known question about the deterministic complexity of maximal independent set [FOCS'87].

Put together with the results of Ghaffari, Kuhn, and Maus [STOC'17] and Ghaffari, Harris, and Kuhn [FOCS'18], we get a general distributed derandomization result that implies  $P\text{-RLOCAL} = P\text{-LOCAL}$ . That is, for any distributed problem whose solution can be checked in polylogarithmic-time, any polylogarithmic-time randomized algorithm can be derandomized to a polylogarithmic-time deterministic algorithm.

By known connections, our result leads also to substantially faster *randomized* algorithms for a number of fundamental problems including  $(\Delta + 1)$ -coloring, MIS, and Lovász Local Lemma.

Through known connections, this general derandomization leads to better *deterministic* and *randomized* distributed algorithms for numerous problems. A sampling of end-results includes **poly( $\log n$ )-round deterministic algorithms for MIS,  $\Delta + 1$  coloring, the Lovász Local Lemma<sup>3</sup>, hypergraph splitting, and defective coloring.** These also lead to substantially improved randomized algorithms, including a poly( $\log \log n$ )-time randomized  $\Delta + 1$  coloring [CLP18] and a poly( $\log \log n$ )-time randomized algorithm for Lovász Local Lemma in constant degree graphs [GHK18].



Use all edges of  $H$   
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## Further Results in the *Active* SUPPORTED Model

- Connection to SLOCAL model [Ghaffari et al., STOC 2017]
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Best LOCAL algorithm:

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Also works in *passive* model:  
 $SLOCAL(t) \rightarrow SUPPORTED(\Delta^{O(t)})$

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Best LOCAL algorithm:  
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- Set of size  $(\alpha(G) - \epsilon)n$  in  $O(\log_{1+\epsilon} n)$ , respectively  $(1+\epsilon)$  approximation if maximum degree  $\Delta$  constant
- Cannot be approximated by  $o(\Delta / \log \Delta)$  in time  $o(\log_{\Delta} n)$  in the active SUPPORTED model

## Bigger Open Question/Opportunity

How to efficiently leverage such preprocessing/distributed computing to efficiently scale controllers (and network updates)?

So far largely unexplored

So let's get back things we know about 😊  
Congestion and network functions?



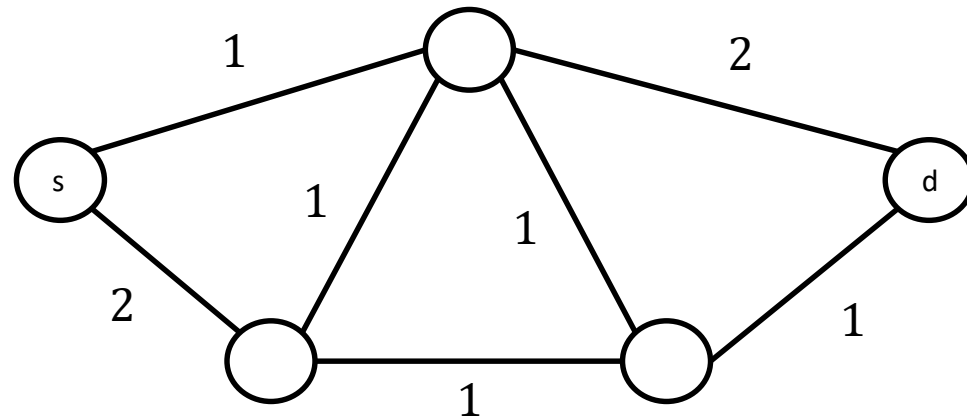


## Congestion?

- “Stronger” consistency constraint: also do not violate link capacities

## Congestion?

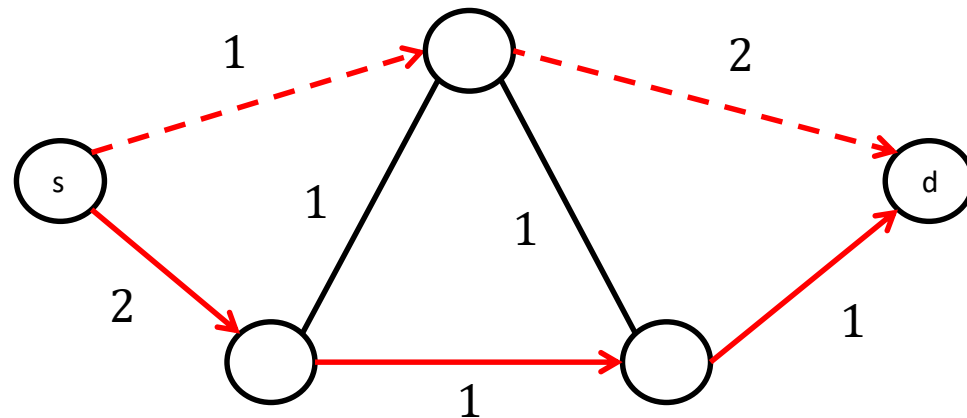
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## Congestion?

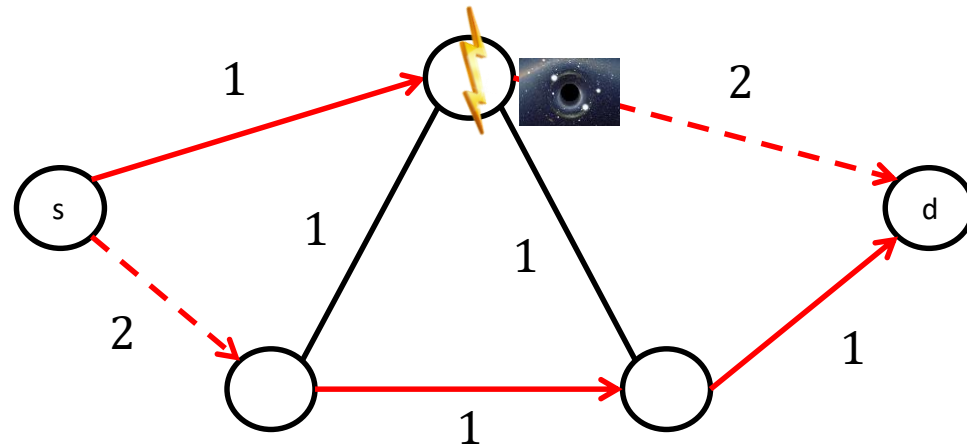
- “Stronger” consistency constraint: also do not violate link capacities
  - Flow size: 1



Round 0

## Congestion?

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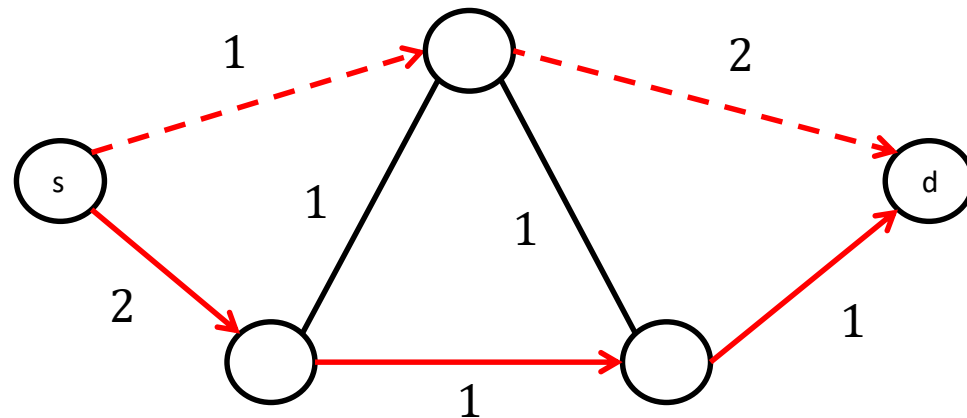


Round 1



## Congestion?

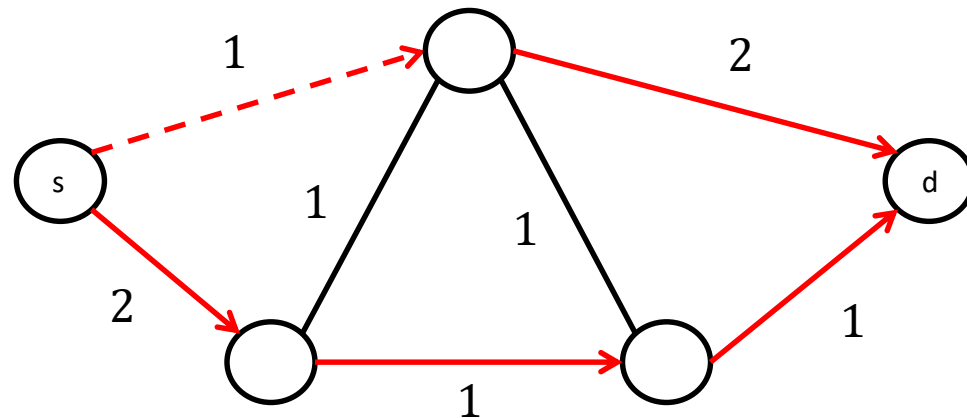
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Round 0

## Congestion?

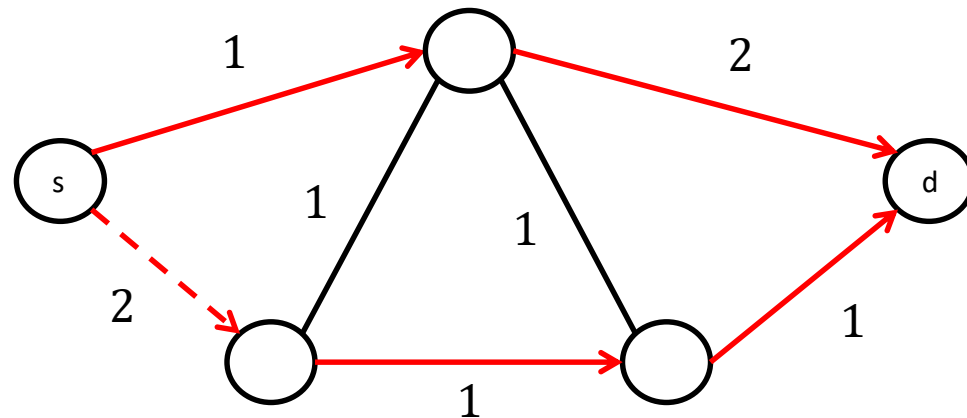
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Round 1

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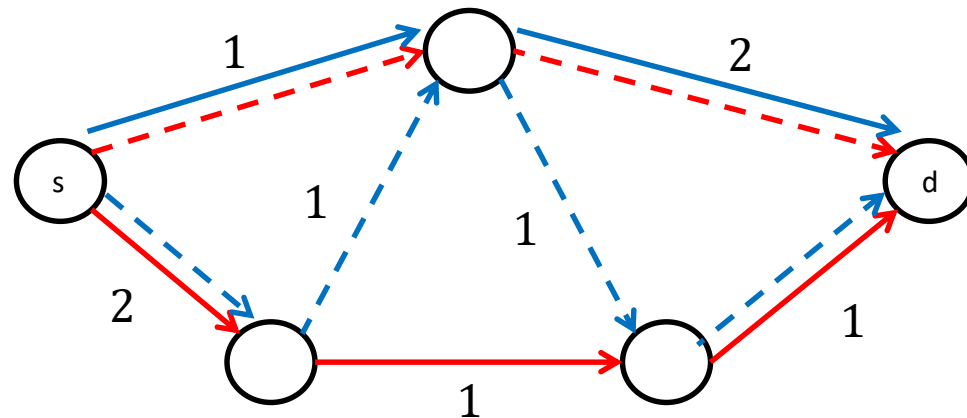
- “Stronger” consistency constraint: also do not violate link capacities
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## Round 2

## Congestion?

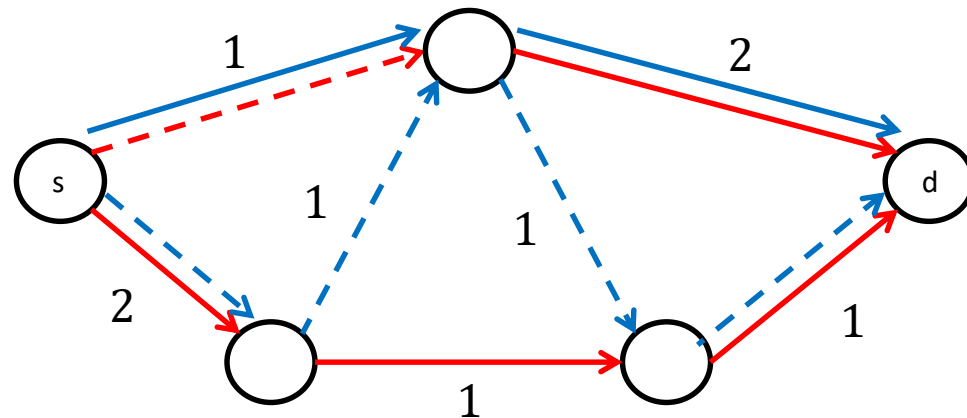
- “Stronger” consistency constraint: also do not violate link capacities
  - Flow size: 1, 1



Round 0

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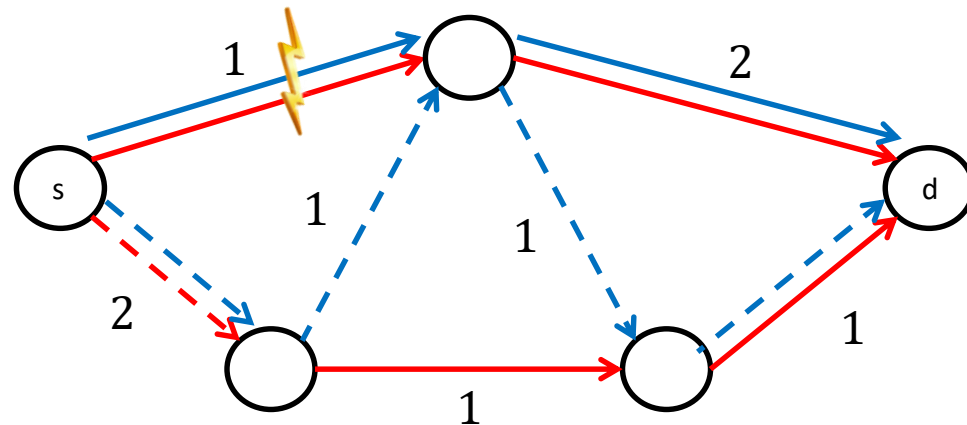
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Round **1**

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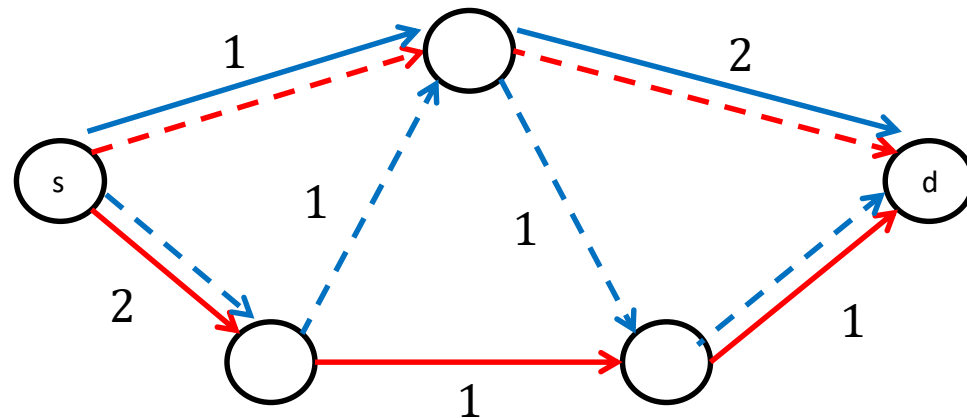
- “Stronger” consistency constraint: also do not violate link capacities
  - Flow size: 1, 1



Round 2

## Congestion?

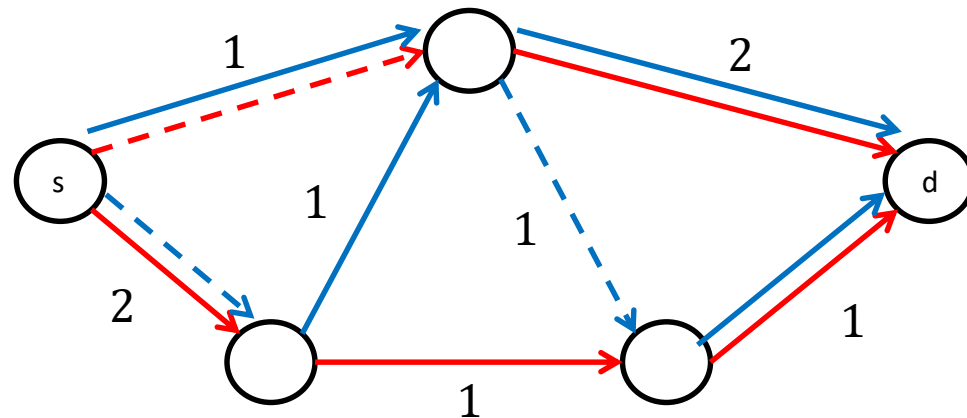
- “Stronger” consistency constraint: also do not violate link capacities
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Round 0

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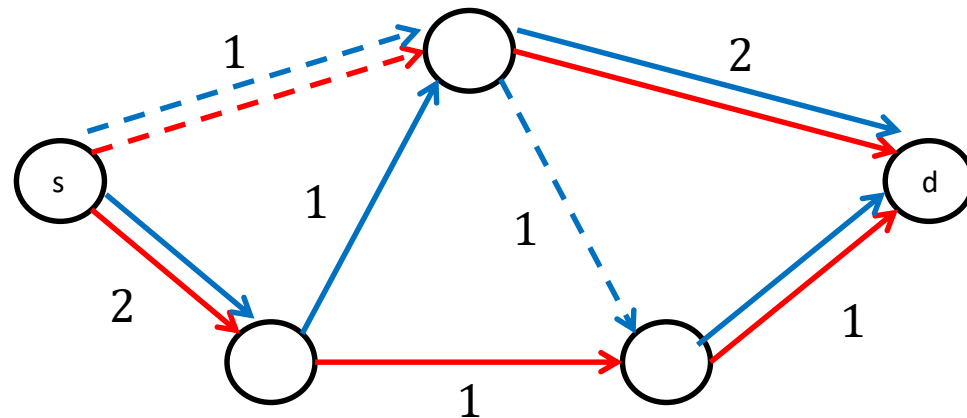


Round **1**



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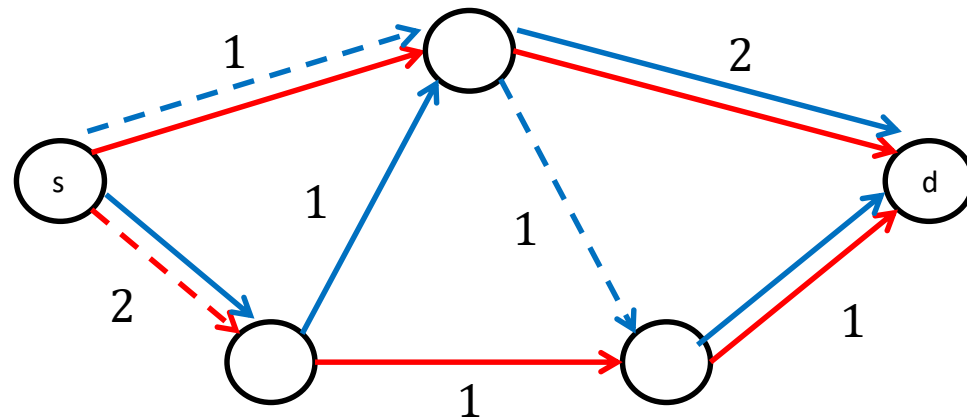
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## Round 2

## Congestion?

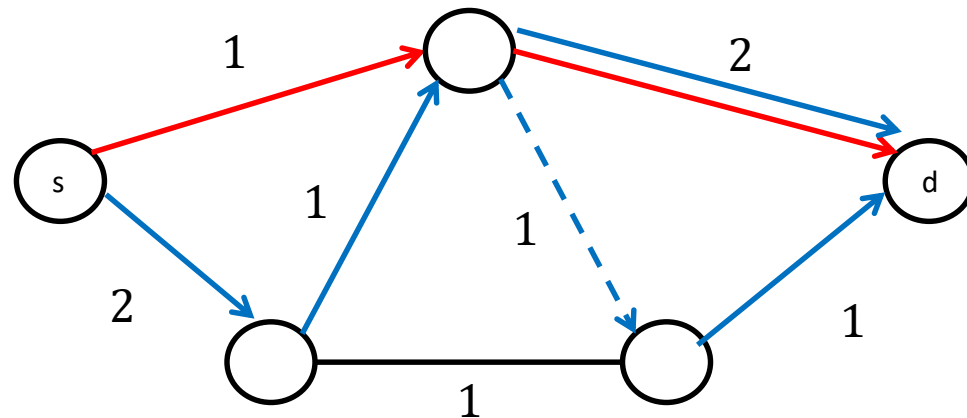
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## Round 3

## Congestion?

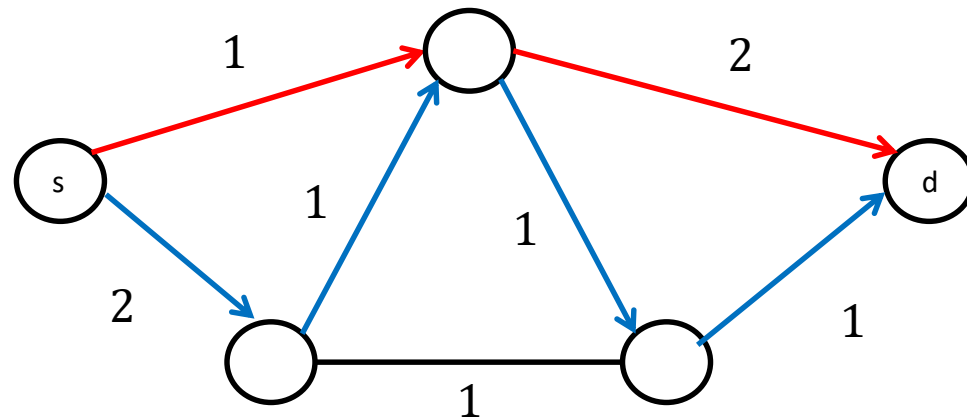
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### Round 3

## Congestion?

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## Round 4

## Complexity of Avoiding Congestion?

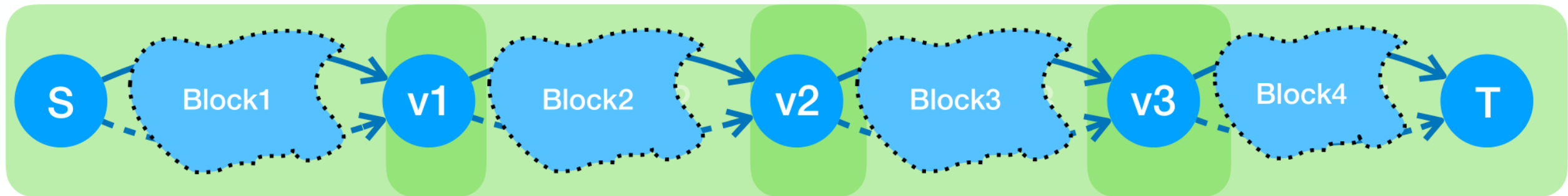
- NP-hard already for 2 unit size flows on general graphs
- Also NP-hard on acyclic graphs for  $k$  flows
  - But can be FPT characterized for  $k$  flows on acyclic graphs:  $O(2^{O(k \log k)} |G|)$ 
    - In other words, linear runtime for constant  $k$  on DAGs
- For just 2 unit size flows (where old/new ***individually*** is a DAG): Optimal schedule in P (NPH for 6)

*Congestion-Free Rerouting of Flows on DAGs.* S. Akhoondian Amiri, S. Dudycz, S. Schmid, S. Widerrecht, ICALP'18

*On Polynomial-Time Congestion-Free Software-Defined Network Updates.* AA, D., M. Parham, S., S. W., Networking'19

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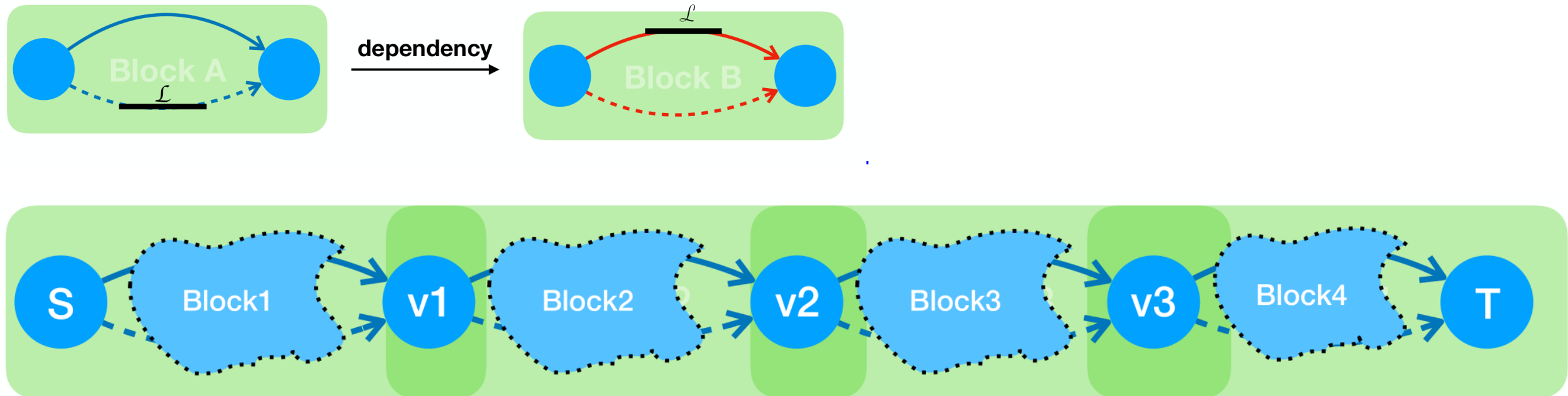
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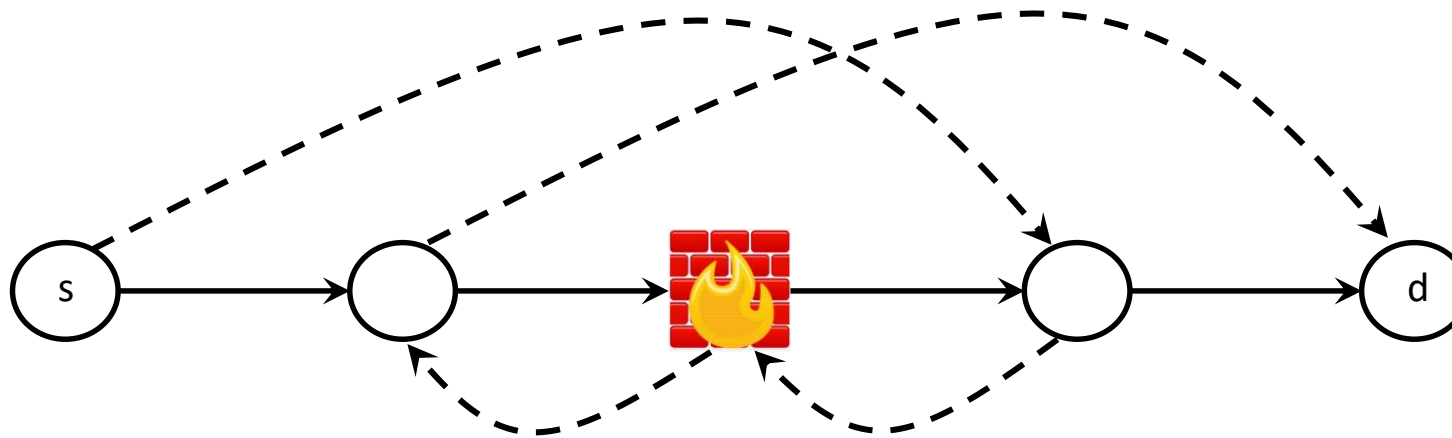


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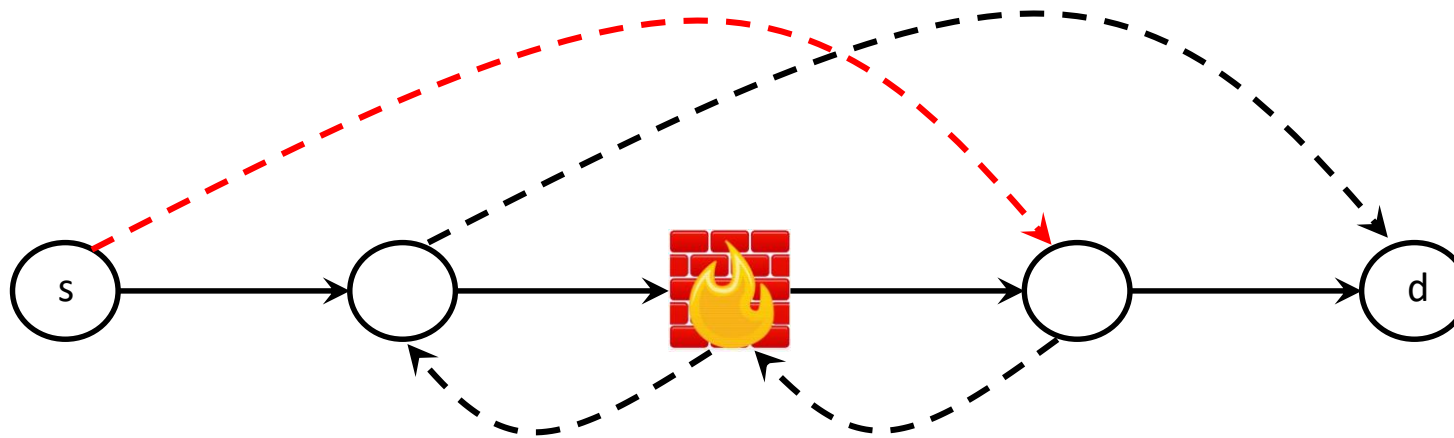


## Take a Step Back: No Loops and a Firewall

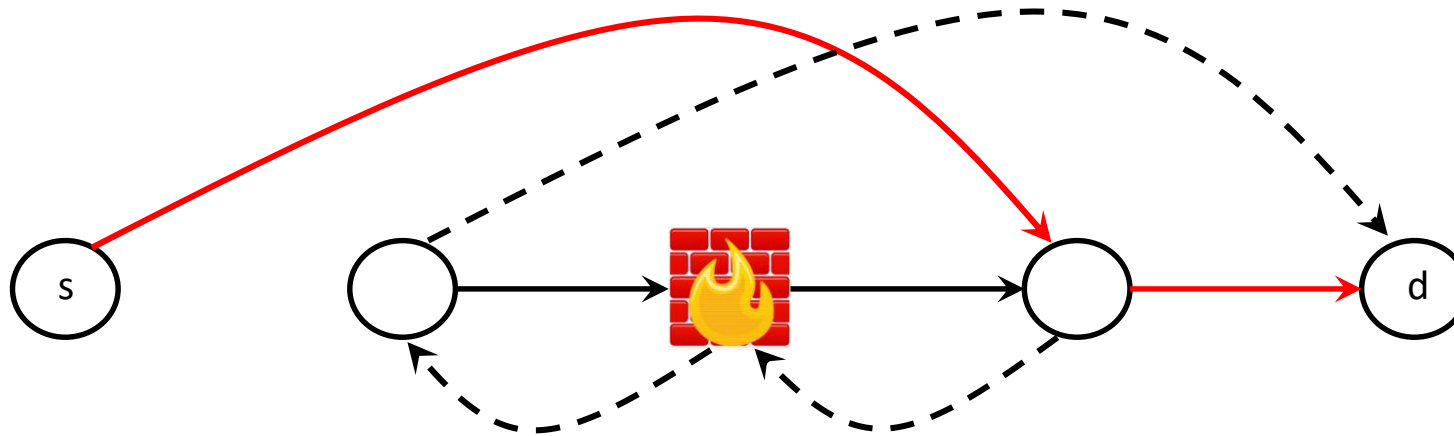
Which forwarding rule to update first?



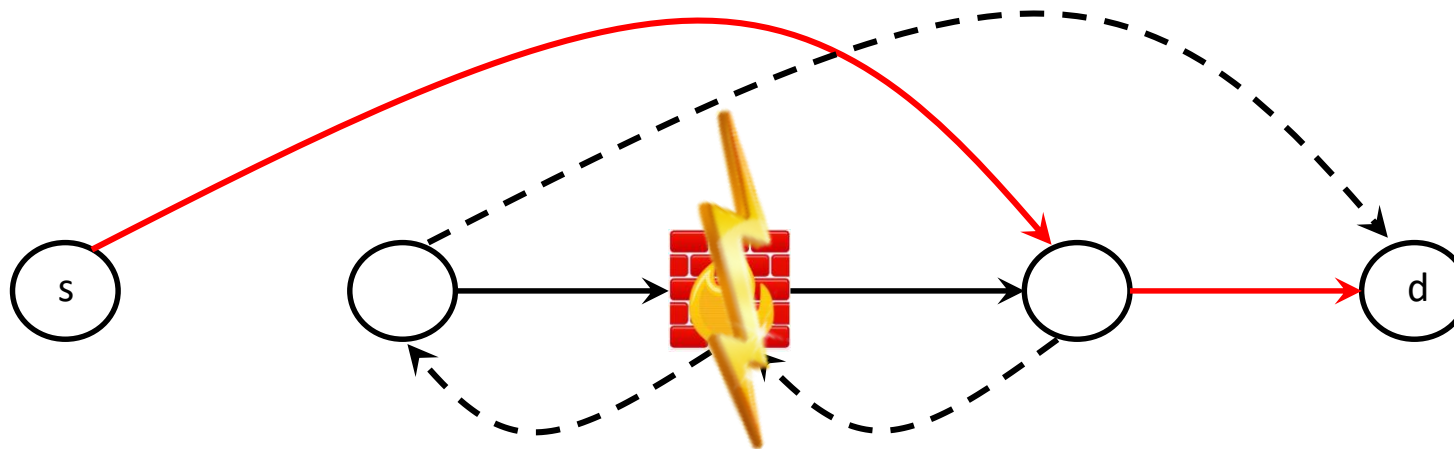
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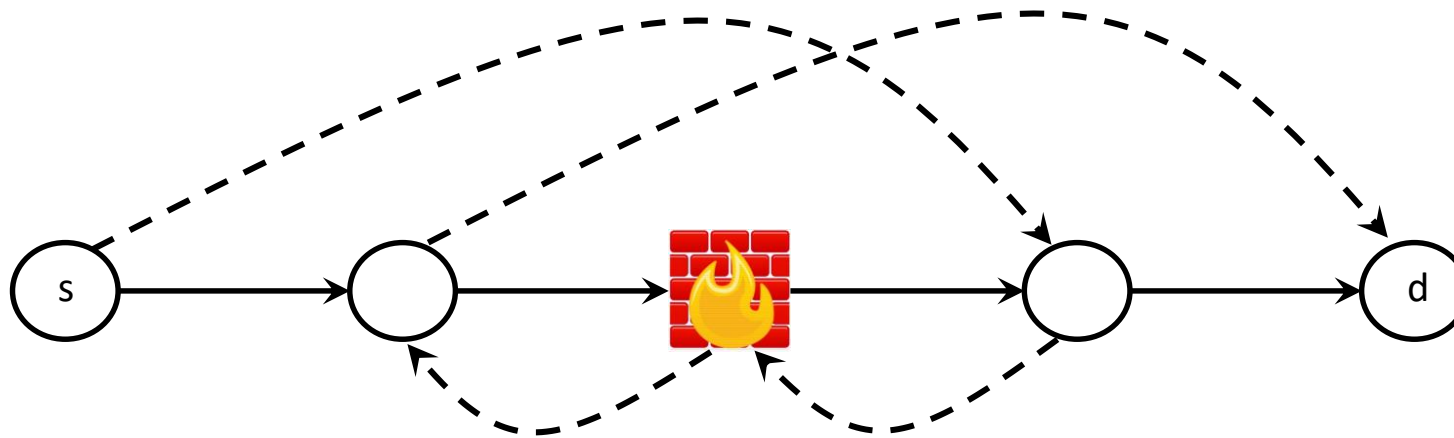
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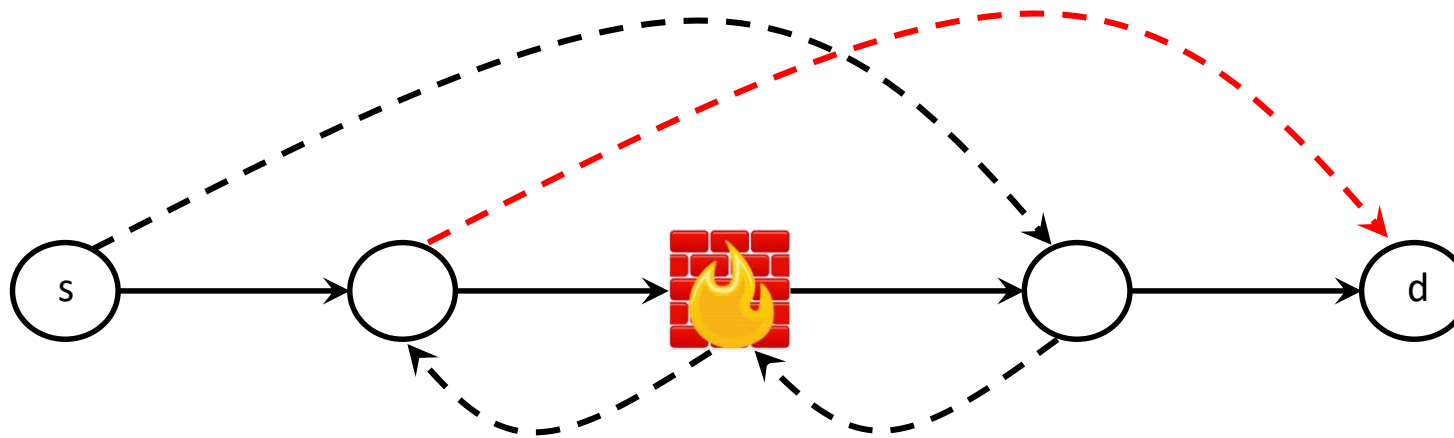
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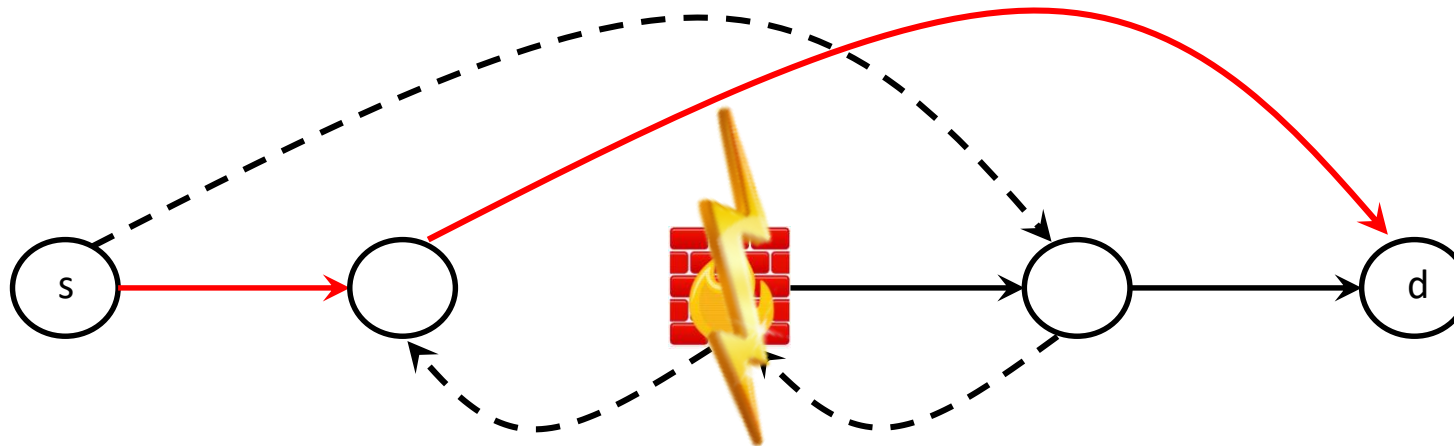
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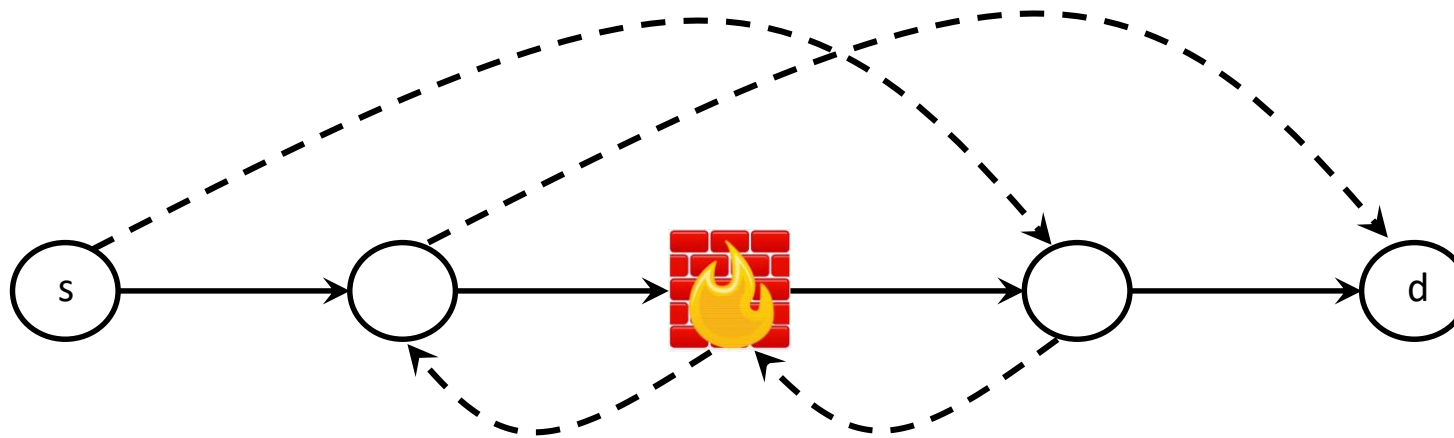
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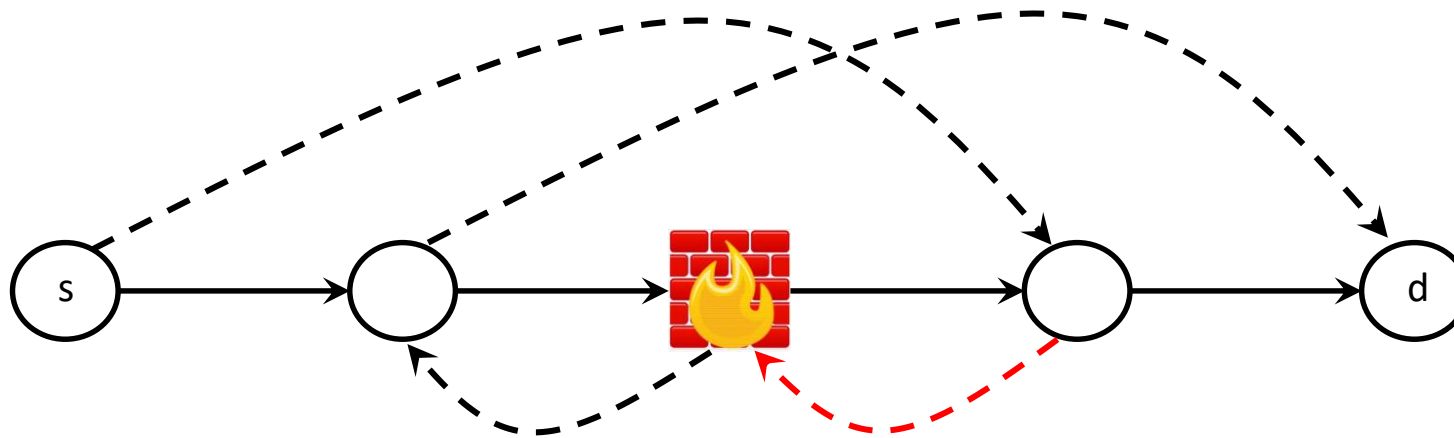


## Take a Step Back: No Loops and a Firewall

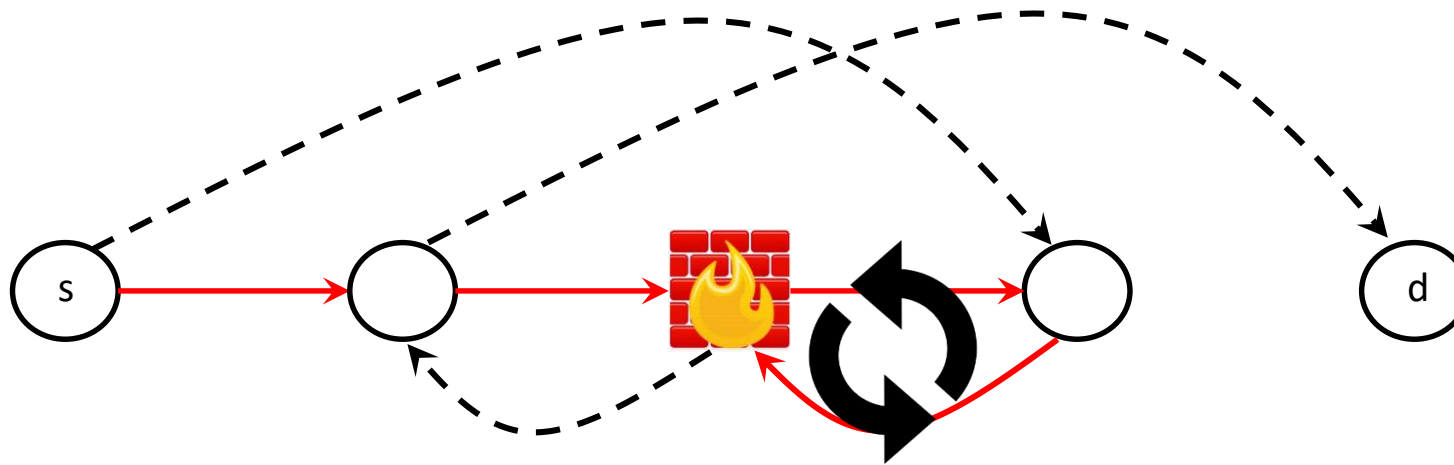




## Take a Step Back: No Loops and a Firewall



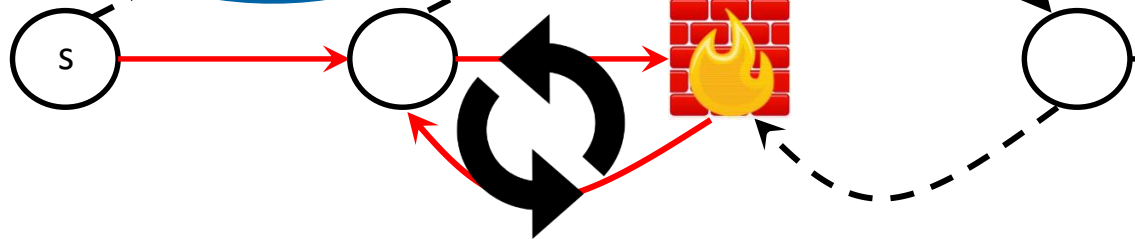
## Take a Step Back: No Loops and a Firewall



## Take a Step Back: No Loops and a Firewall

However: If packets must either take the new or the old path (and no mix), then polynomial-time solvable (Cerný et al., DISC 2016)

s



```
graph LR; s((s)) -- red --> R(( )); R -- red --> F[Firewall]; F -- red --> R; R -.-> s; R --- D(( )); D --- S(( ))
```

&  can conflict!

Satisfy both  &  ?  
NP-hard!

*Transiently Secure Network Updates.* A. Ludwig, S. Dudycz, M. Rost, S. Schmid. SIGMETRICS 2016.

## Different model: “tagged” Flows

- Identified by a “tag” in the packet header, update via
  - Install new tag’ rules
  - Switch from tag to tag’ at source

## If we move a flow, will there be congestion?

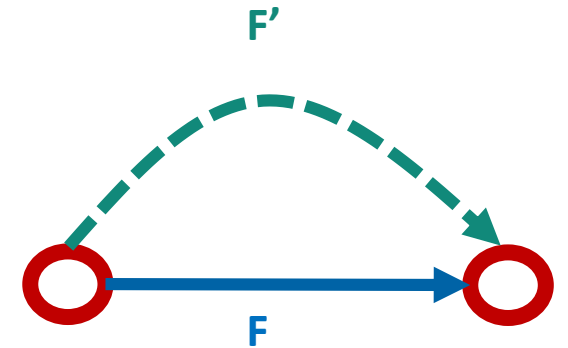
## If we move a flow, will there be congestion?

- How do we move a flow **F**? Usually: 2-phase commit: [Reitblatt et al., SIGCOMM'12]



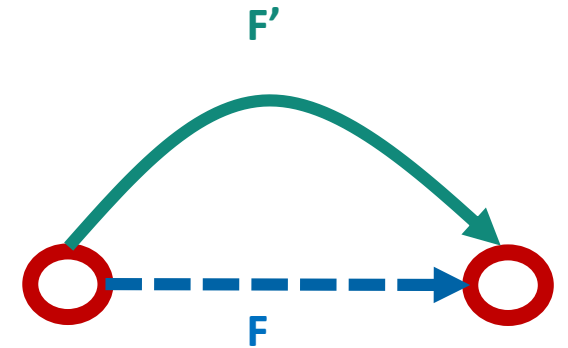
## If we move a flow, will there be congestion?

- How do we move a flow  $F$ ? Usually: 2-phase commit:
  - Deploy new flow rules  $F'$



## If we move a flow, will there be congestion?

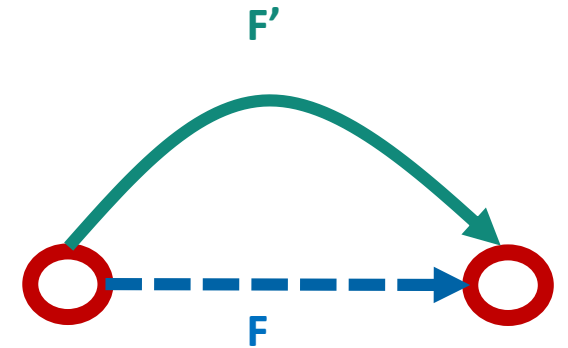
- How do we move a flow **F**? Usually: 2-phase commit:
  - Deploy new flow rules **F'**
  - Change packet tag at source from **F** to **F'**





## If we move a flow, will there be congestion?

- How do we move a flow  $F$ ? Usually: 2-phase commit:
  - Deploy new flow rules  $F'$
  - Change packet tag at source from  $F$  to  $F'$



Can also be implemented by  
proof-labeling techniques

“hand holding”?

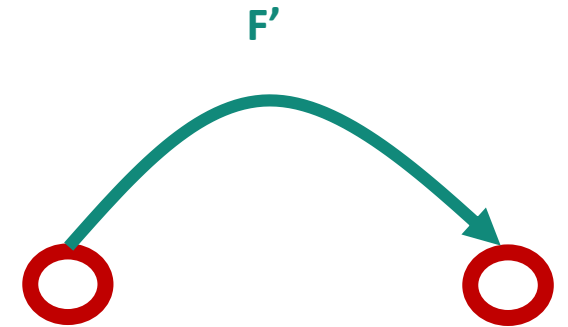
Go backwards with  
distance information

Respects network functions!



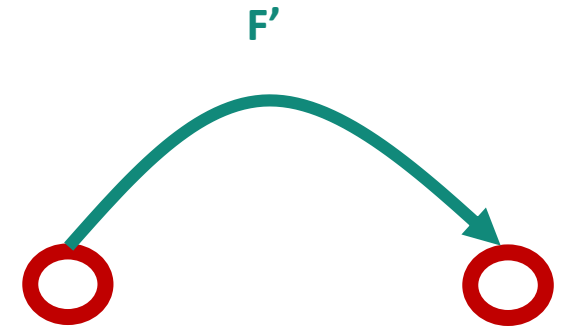
## If we move a flow, will there be congestion?

- How do we move a flow **F**? Usually: 2-phase commit:
  - Deploy new flow rules **F'**
  - Change packet tag at source from **F** to **F'**
  - Clean-up of old rules



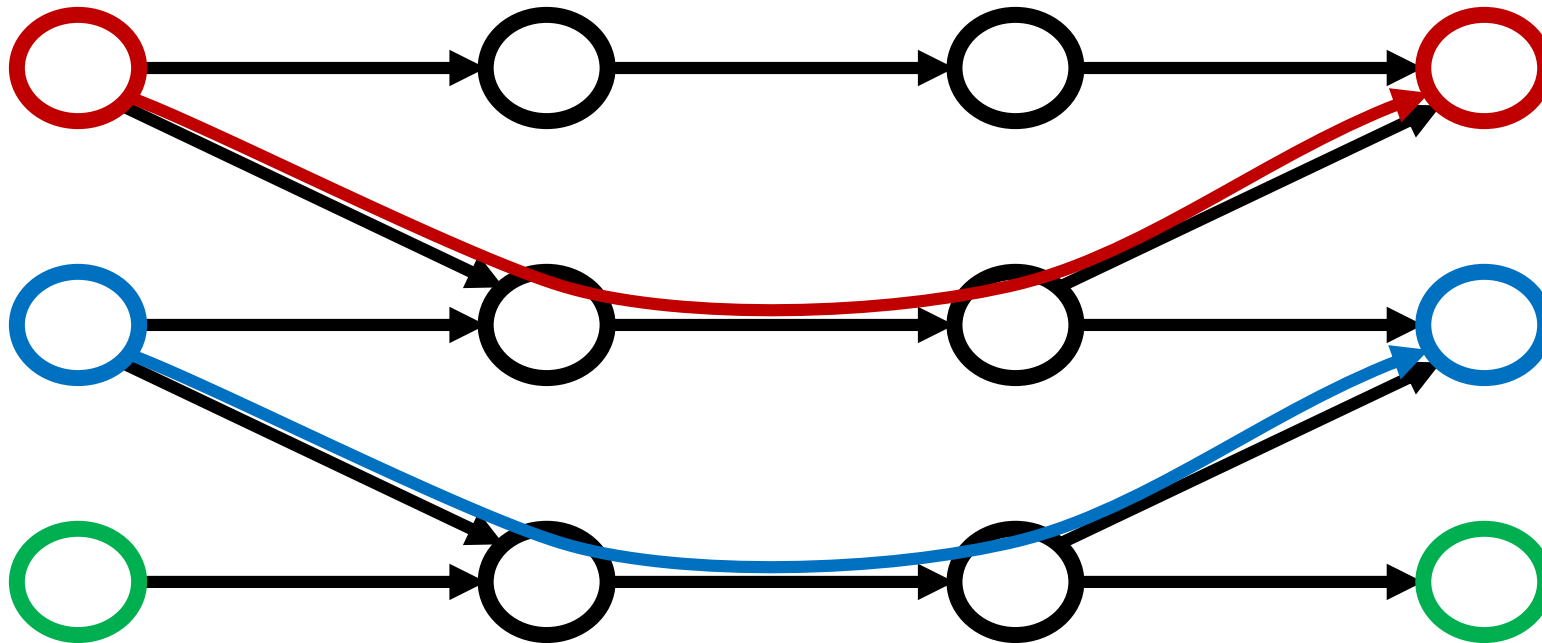
## If we move a flow, will there be congestion?

- How do we move a flow **F**? Usually: 2-phase commit:
  - Deploy new flow rules **F'**
  - Change packet tag at source from **F** to **F'**
  - Clean-up of old rules
  
- First check:
  - Is the new network state without congestion?
  - Easy 😊 (flow size versus capacity)
  
- Is that it?



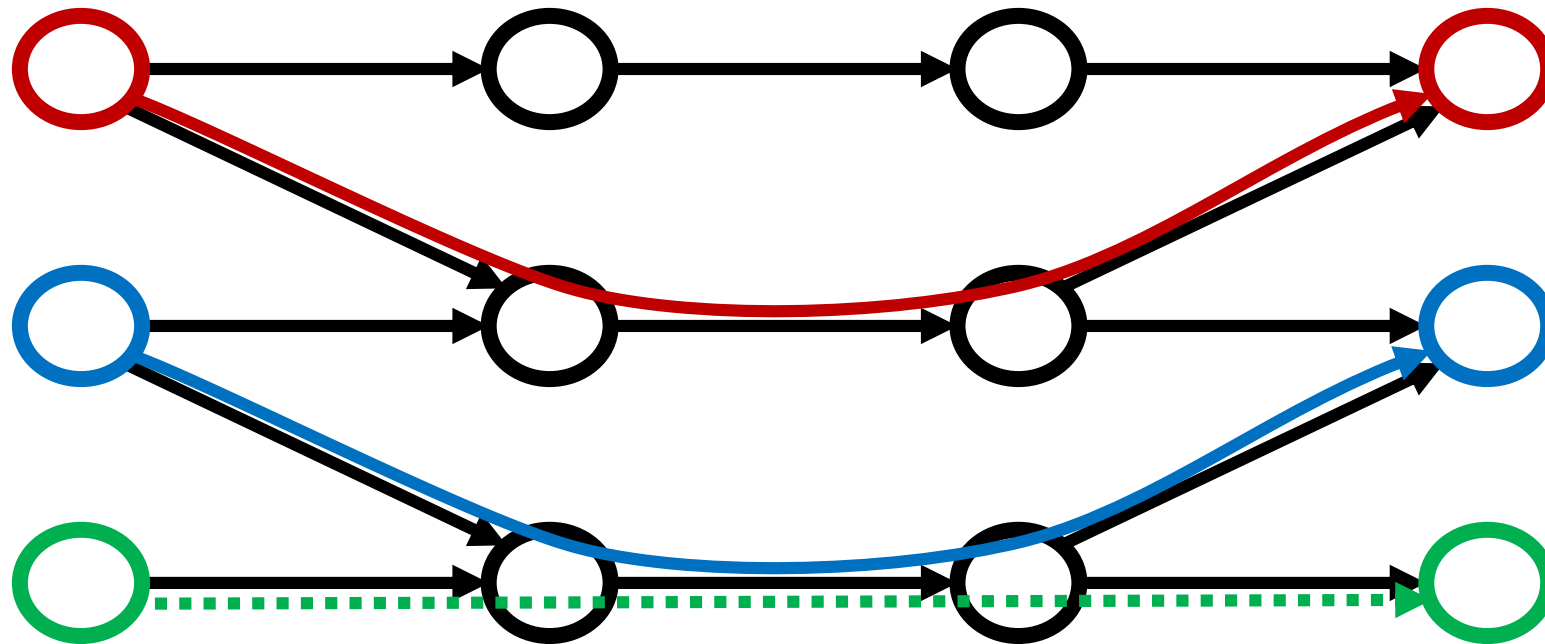
Also verifiable by proof-labeling techniques

## A Small Sample Network



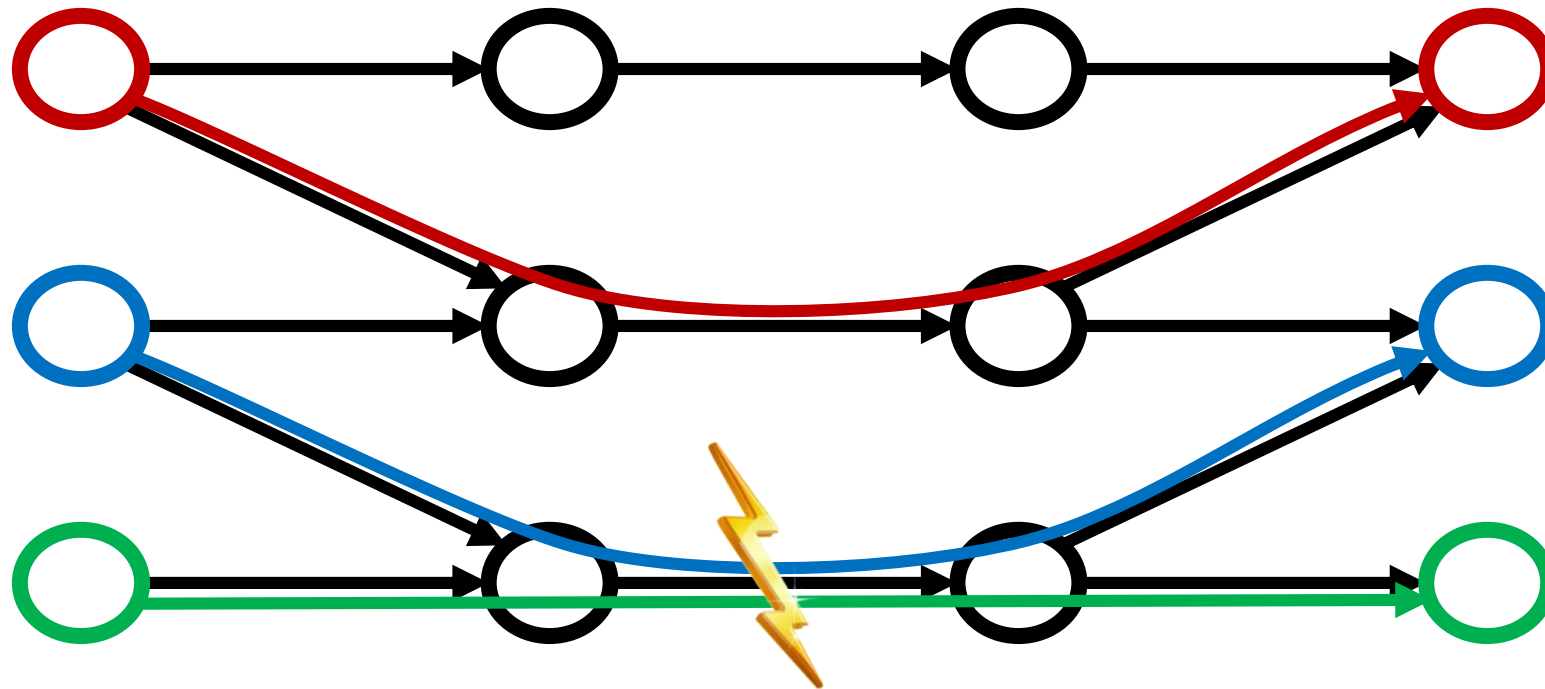
Unit size flows and capacities

## Green wants to send as well



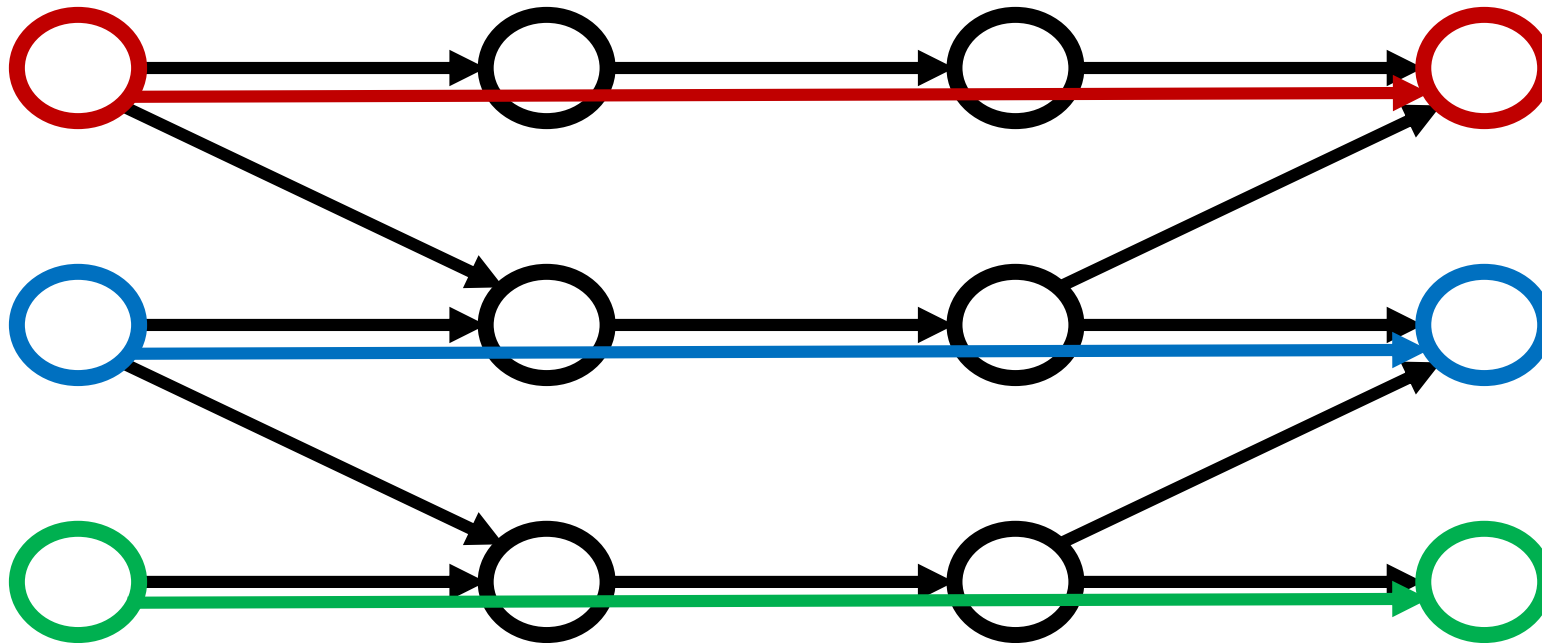
Unit size flows and capacities

## Congestion!



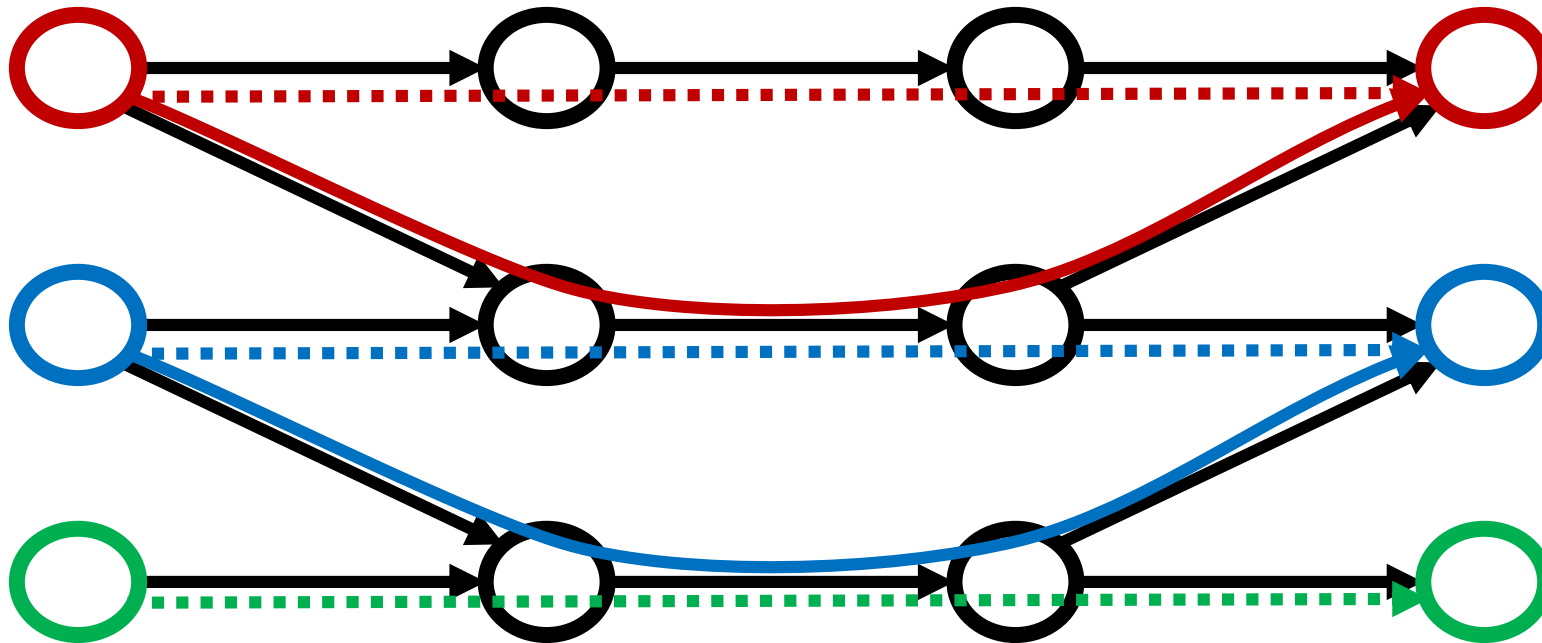
Unit size flows and capacities

## This would work



Unit size flows and capacities

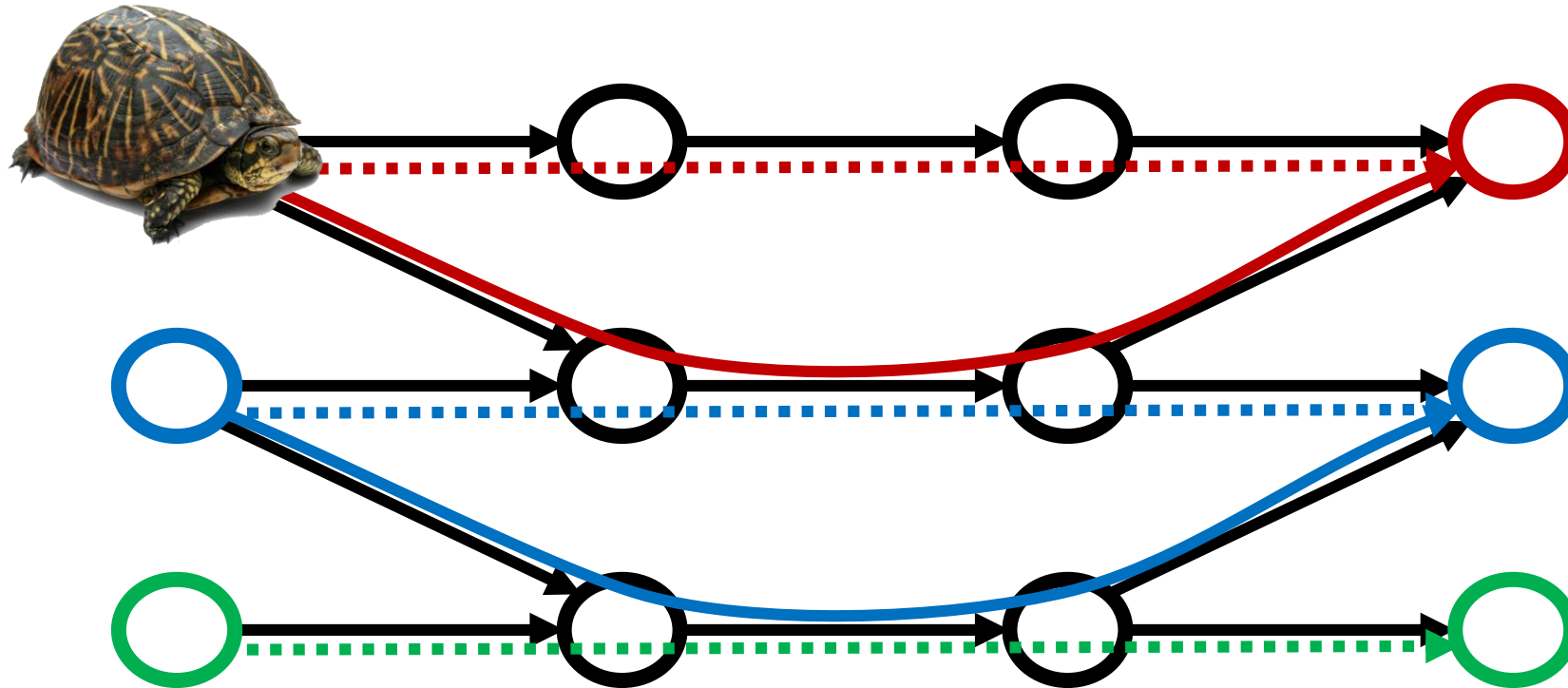
So lets go back



Unit size flows and capacities

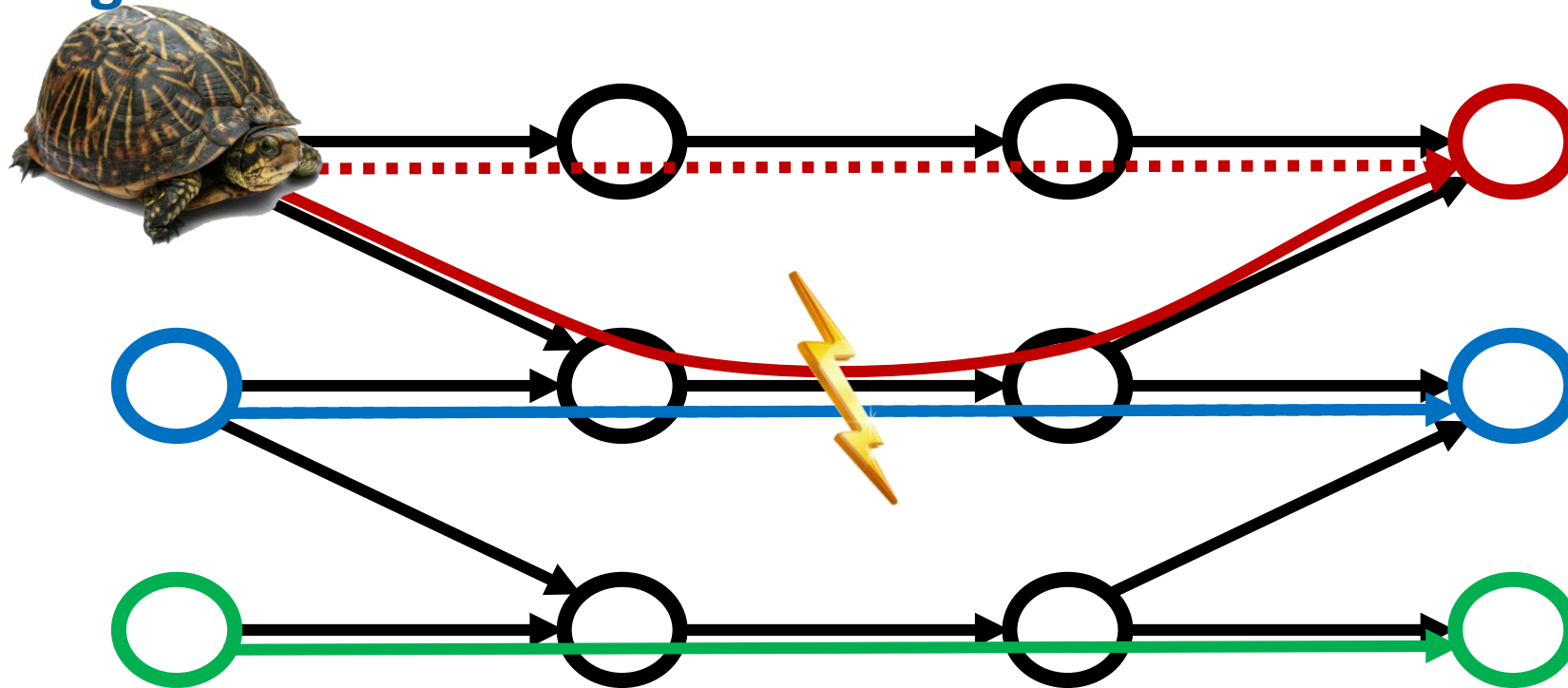


But Red is a bit Slow..



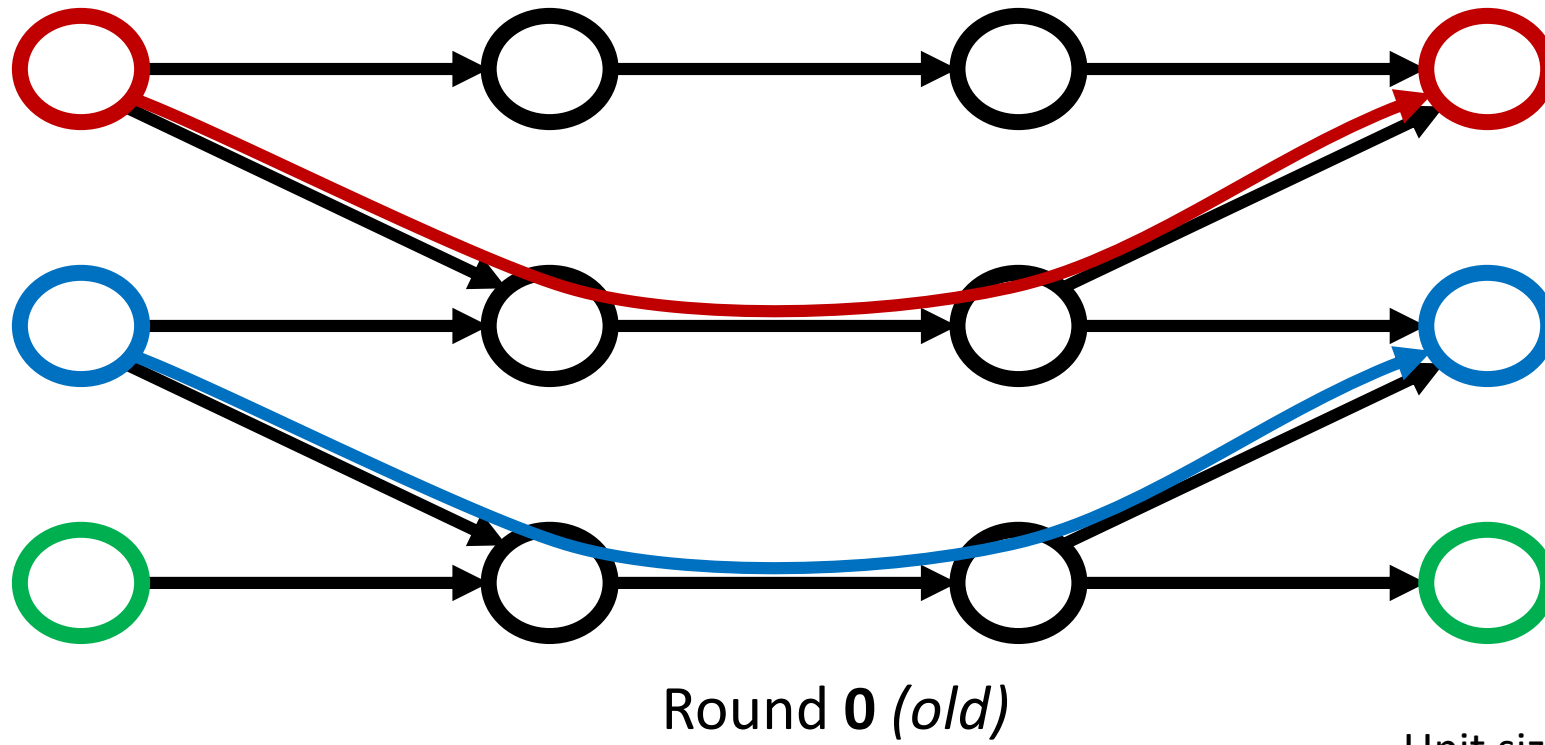
Unit size flows and capacities

## Congestion Again!



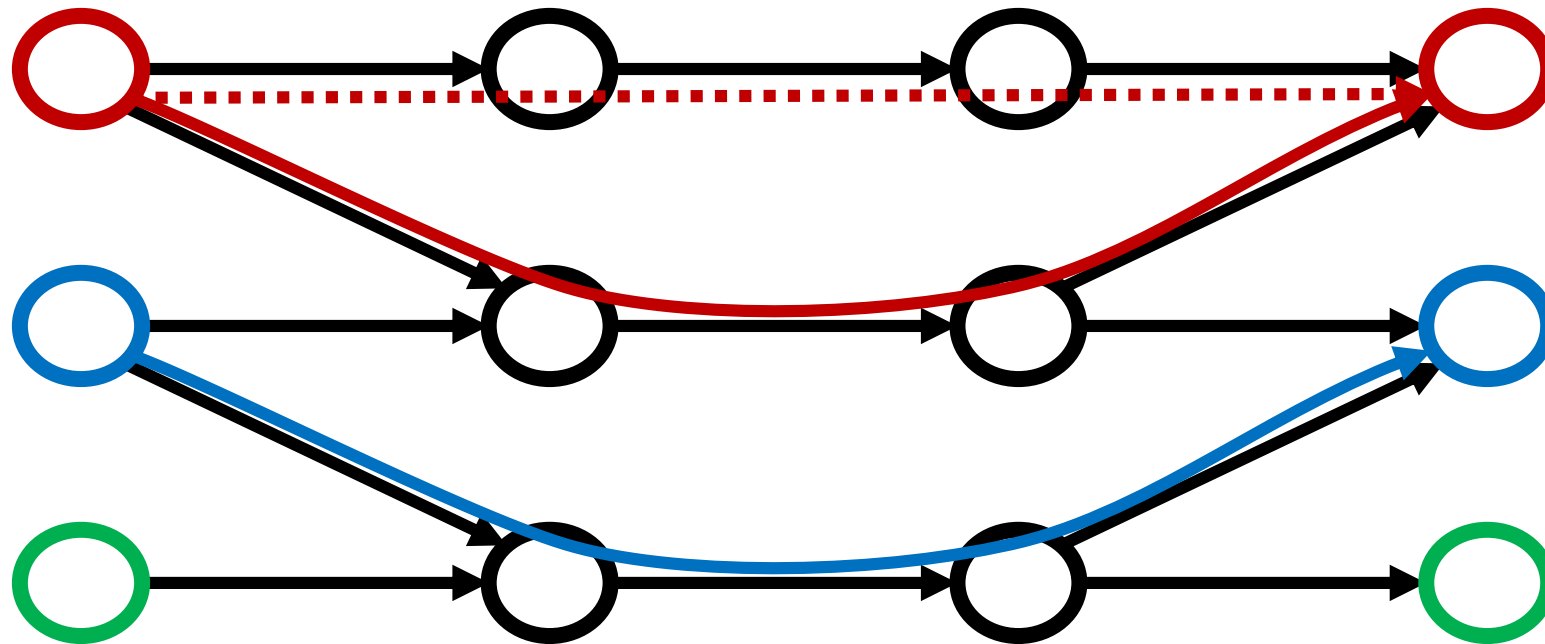
Unit size flows and capacities

So lets go Back ...



Unit size flows and capacities

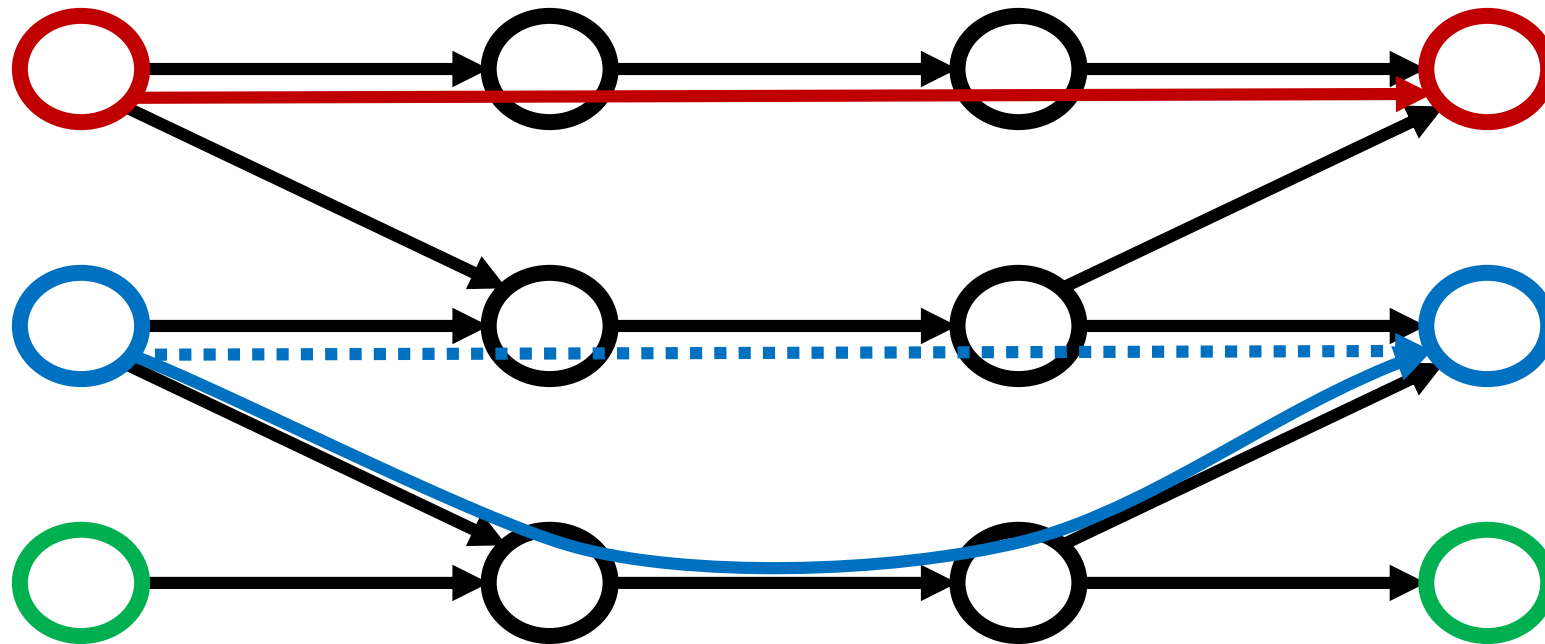
## First, Red switches



Round 1

Unit size flows and capacities

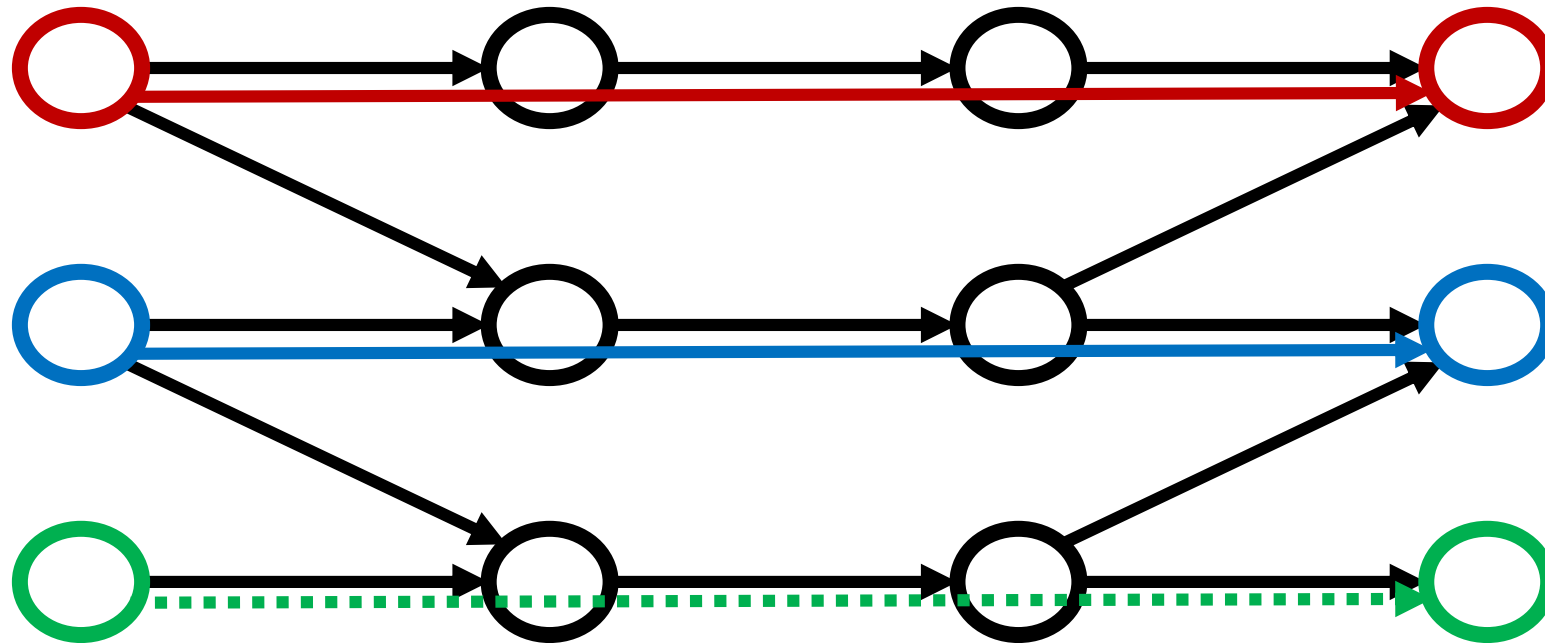
## Then, Blue ...



Round 2

Unit size flows and capacities

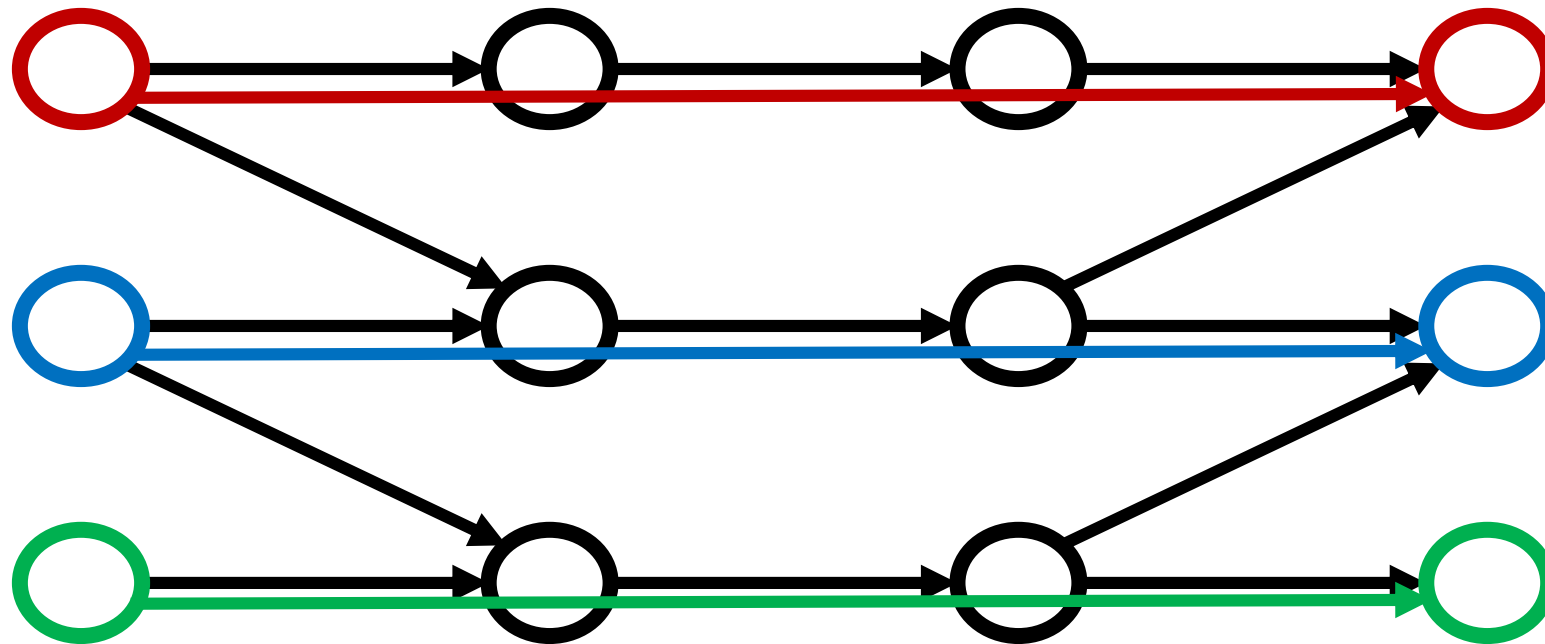
## And then, Green ...



Round 3

Unit size flows and capacities

Done



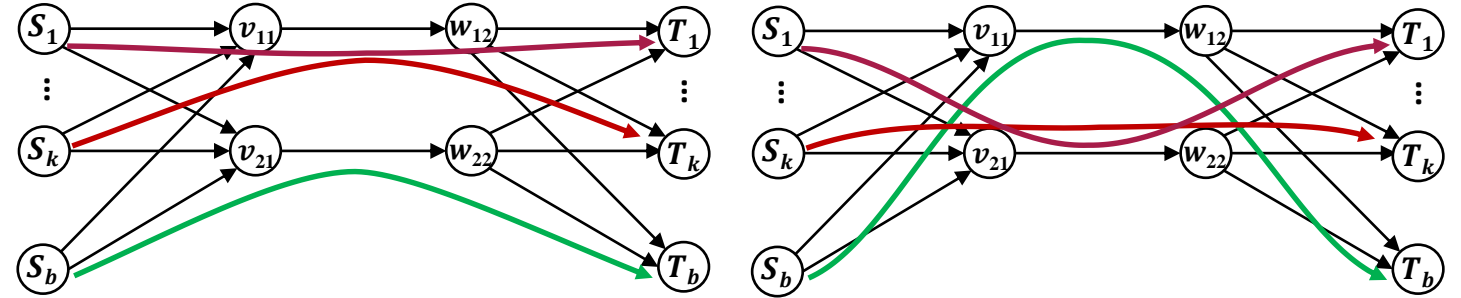
Round 3 (new)

Unit size flows and capacities

## How hard is this (feasibility)?

Flows may only take *old* or *new* paths:

- NP-hard via reduction from Partition





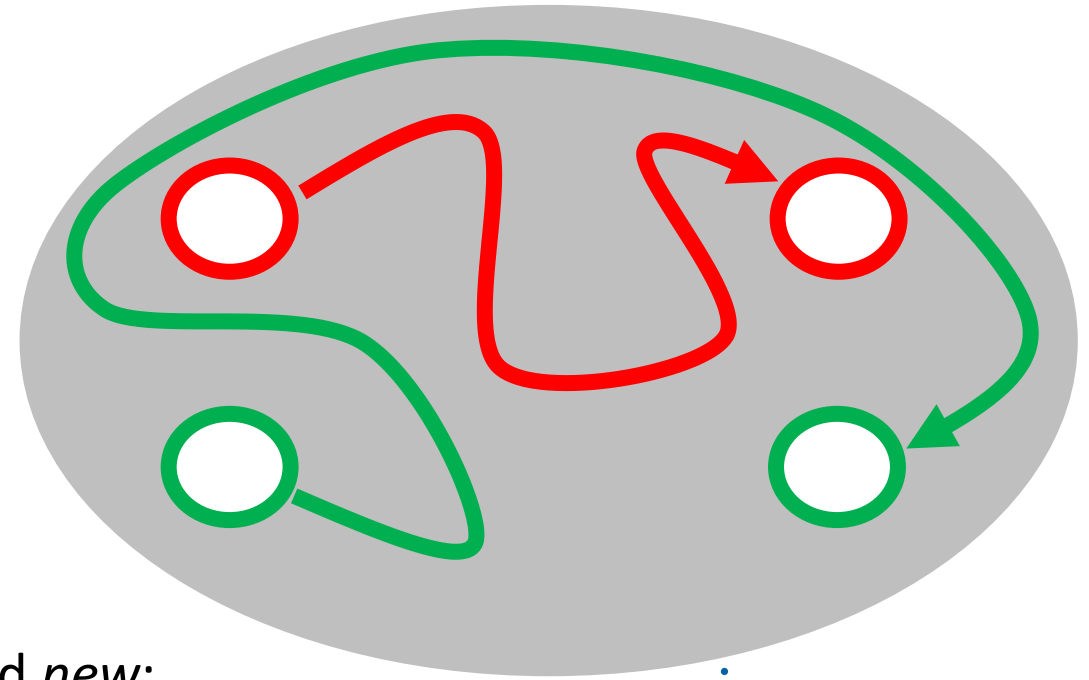
## How hard is this (feasibility)?

Flows may only take *old* or *new* paths:

- NP-hard via reduction from Partition

Intermediate flow allocations not restricted to *old* and *new*:

- NP-hard already for just 2 unit size flows



Hardness intuition: find  
intermediate path for “storage”

*On the Consistent Migration of Unsplittable Flows: Upper and Lower Complexity Bounds (Foerster, NCA 2017)*

## How hard is this (feasibility)?

Flows may only take *old* or *new* paths:

- NP-hard via reduction from Partition

Intermediate flow allocations not restricted to *old* and *new*:

- NP-hard already for just 2 unit size flows

- Is the problem at least in NP? .

Some flows might need to move  
back and forth repeatedly° ☹️

*On the Consistent Migration of Unsplittable Flows: Upper and Lower Complexity Bounds (Foerster, NCA 2017)*

## How hard is this (feasibility)?

Flows may only take *old* or *new* paths:

- NP-hard via reduction from Partition

Intermediate flow allocations not restricted to *old* and *new*:

- NP-hard already for just 2 unit size flows

Not clear if the problem is in NP! (It is known to be in EXPTIME)



How about *splittable* flows?

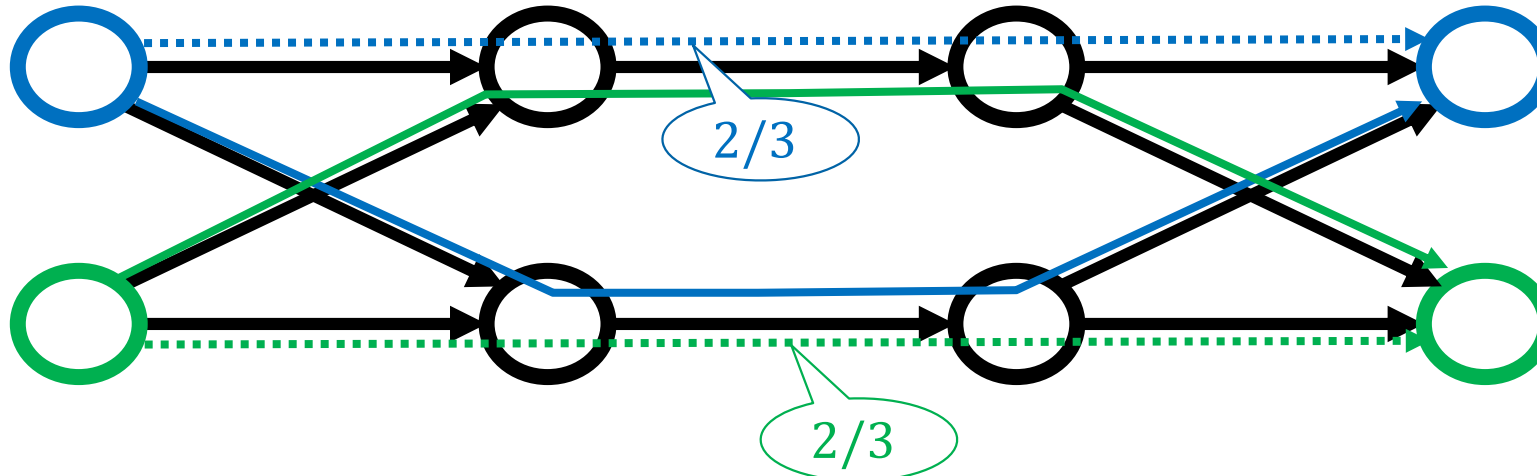
*On the Consistent Migration of Unsplittable Flows: Upper and Lower Complexity Bounds* (Foerster, NCA 2017)

## Consistent Migration of Splittable Flows

Idea: Flows can be on the **old** or **new** route w.r.t. an update

For all edges:  $\sum_{\forall F} \max(\text{old}, \text{new}) \leq \text{capacity}$

*No ordering exists ( $2/3 + 2/3 > 1$ )*

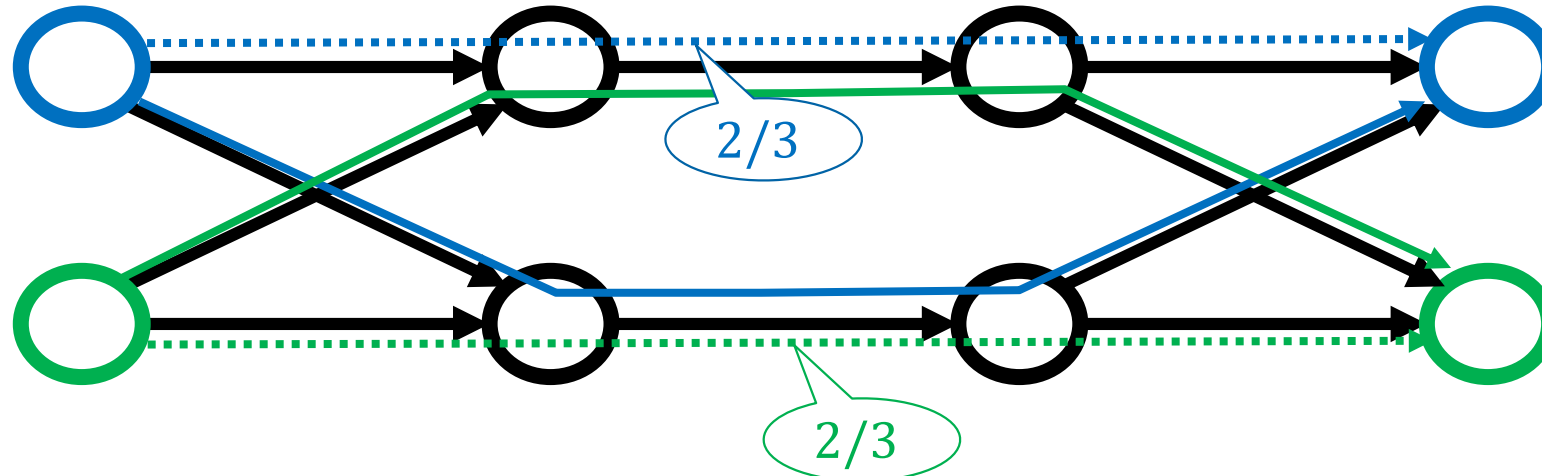


## Consistent Migration of Splittable Flows

Approach of *SWAN*\*: use slack  $x$  (i.e., %)

Here  $x = 1/3$

Move slack  $x \Rightarrow \lceil 1/x \rceil - 1$  staged partial moves



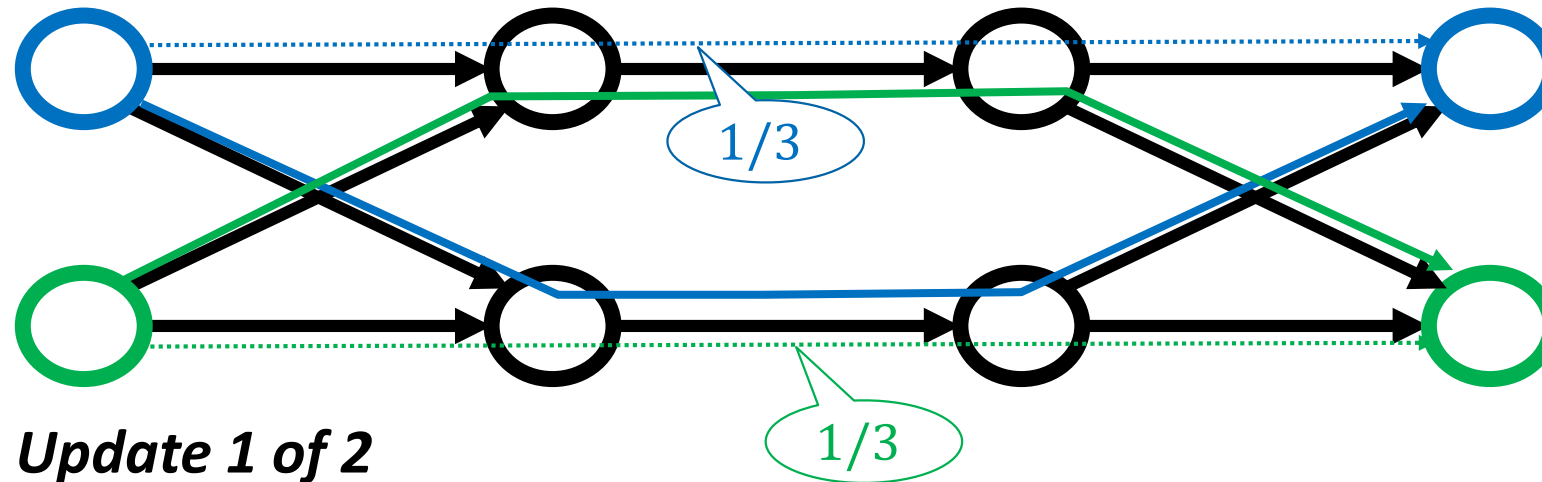
\*: Achieving High Utilization with Software-Driven WAN, SIGCOMM 2013

## Consistent Migration of Splittable Flows

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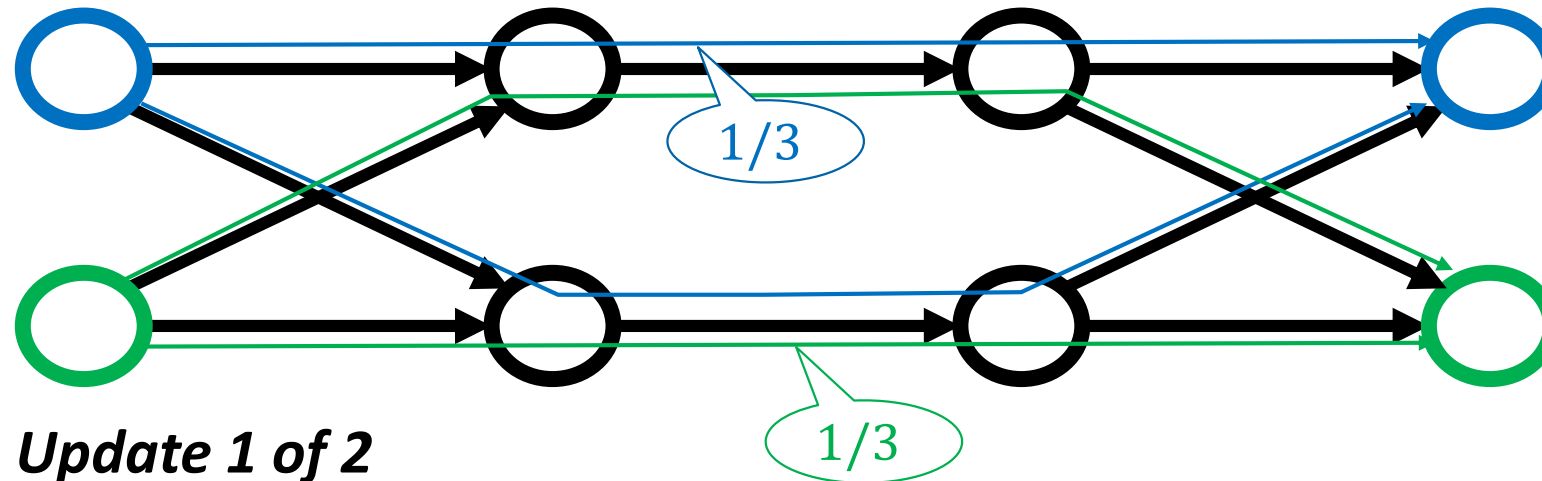


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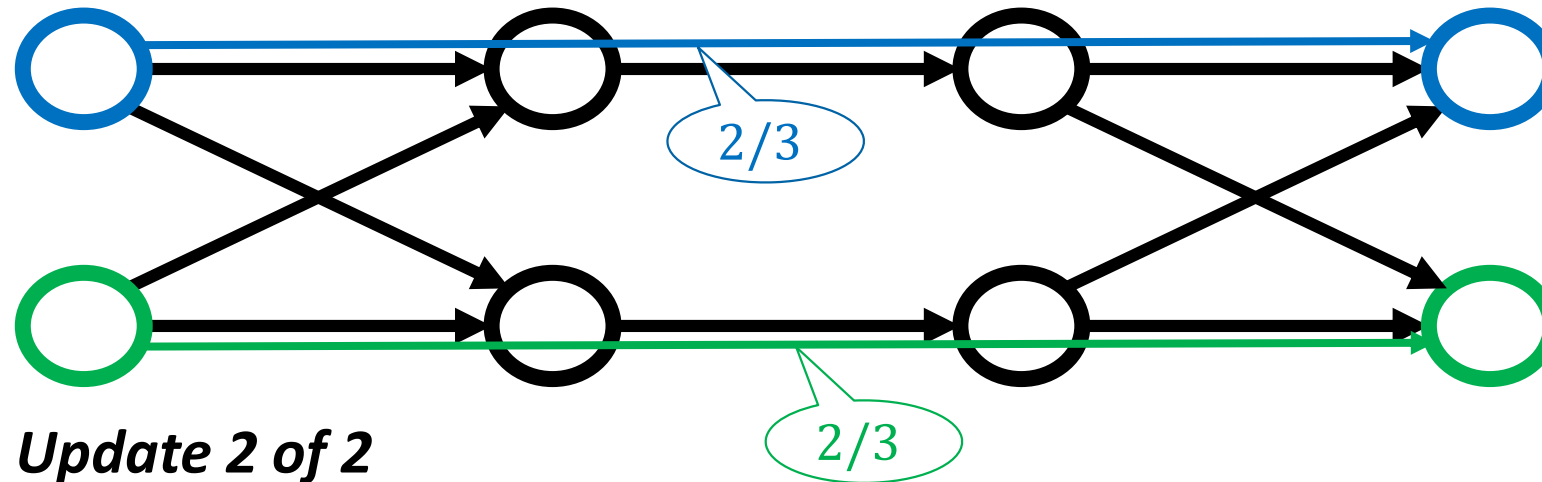


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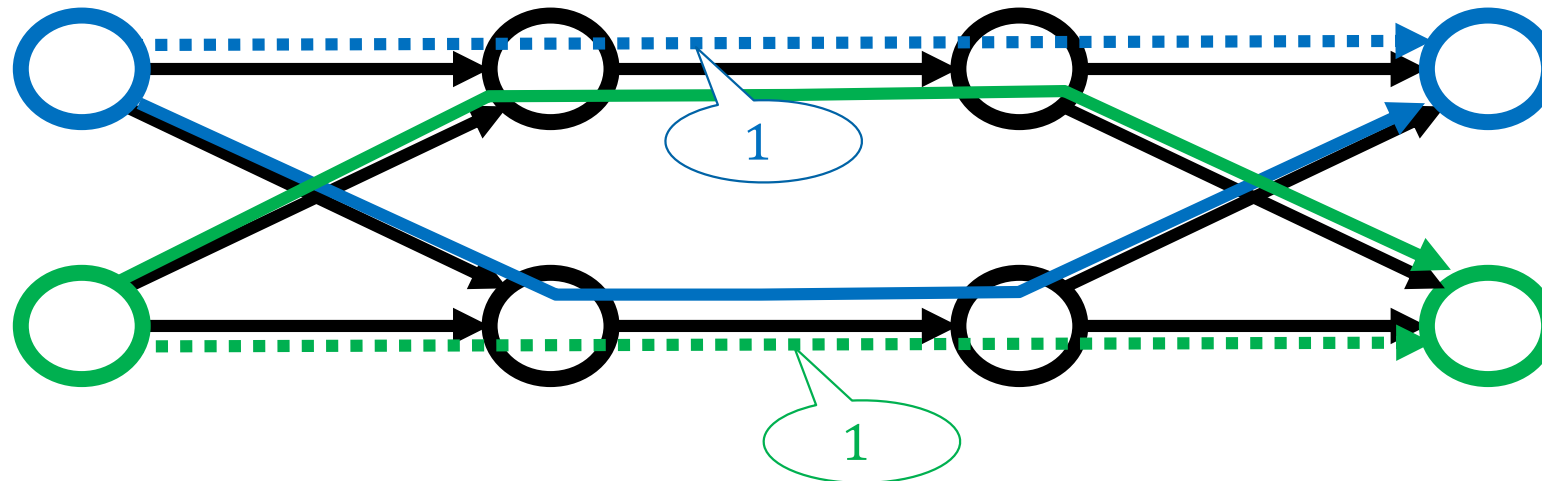
Move slack  $x \Rightarrow \lceil 1/x \rceil - 1$  staged partial moves





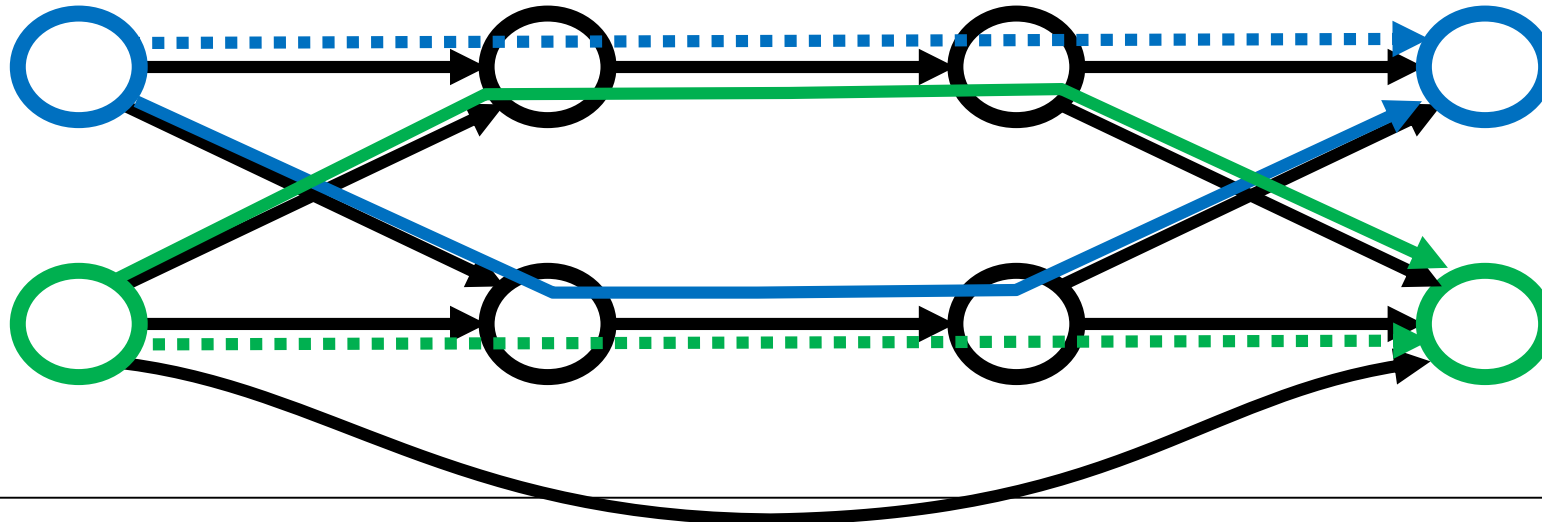
## Consistent Migration of Splittable Flows

No slack on flow edges?



## Consistent Migration of Splittable Flows

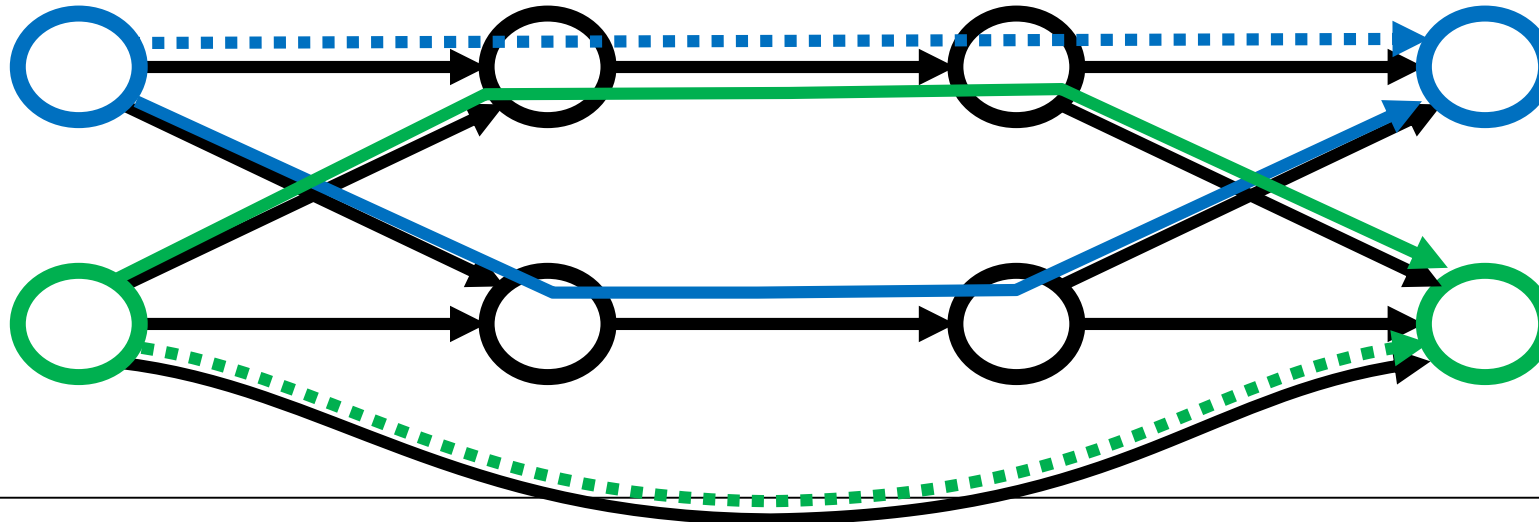
Alternate routes?



## Consistent Migration of Splittable Flows

Think: variable swapping of  $b$  &  $g$

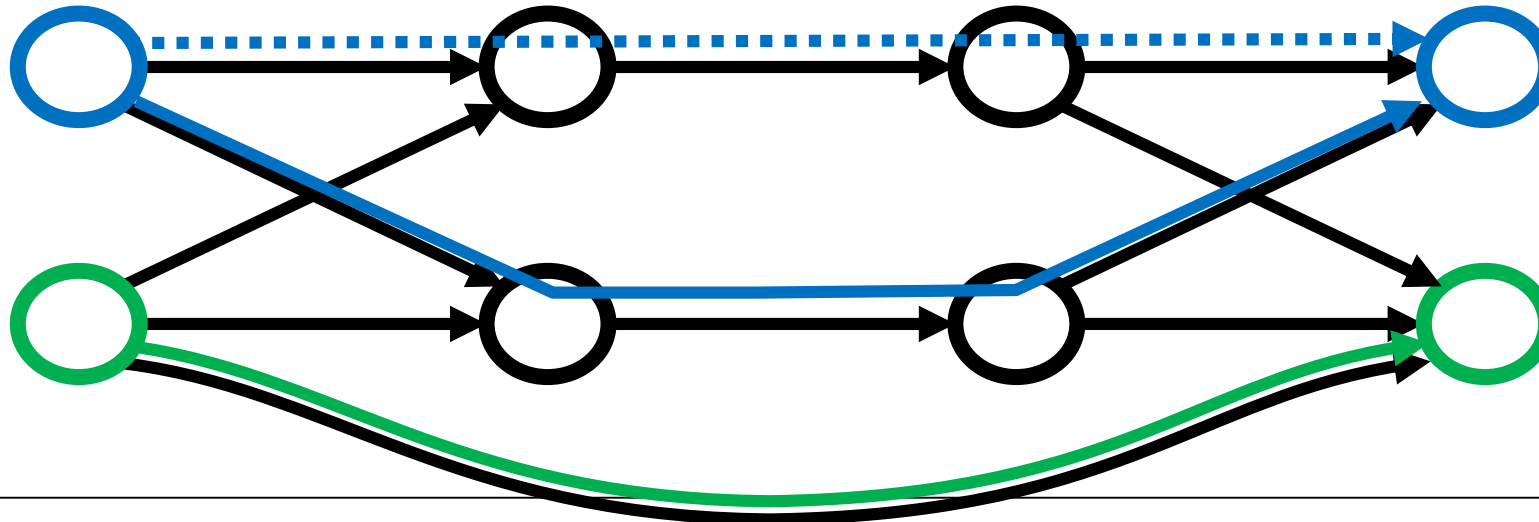
1.  $x := b$ , 2.  $b := g$ , 3.  $g := x$



## Consistent Migration of Splittable Flows

Think: variable swapping of  $b$  &  $g$

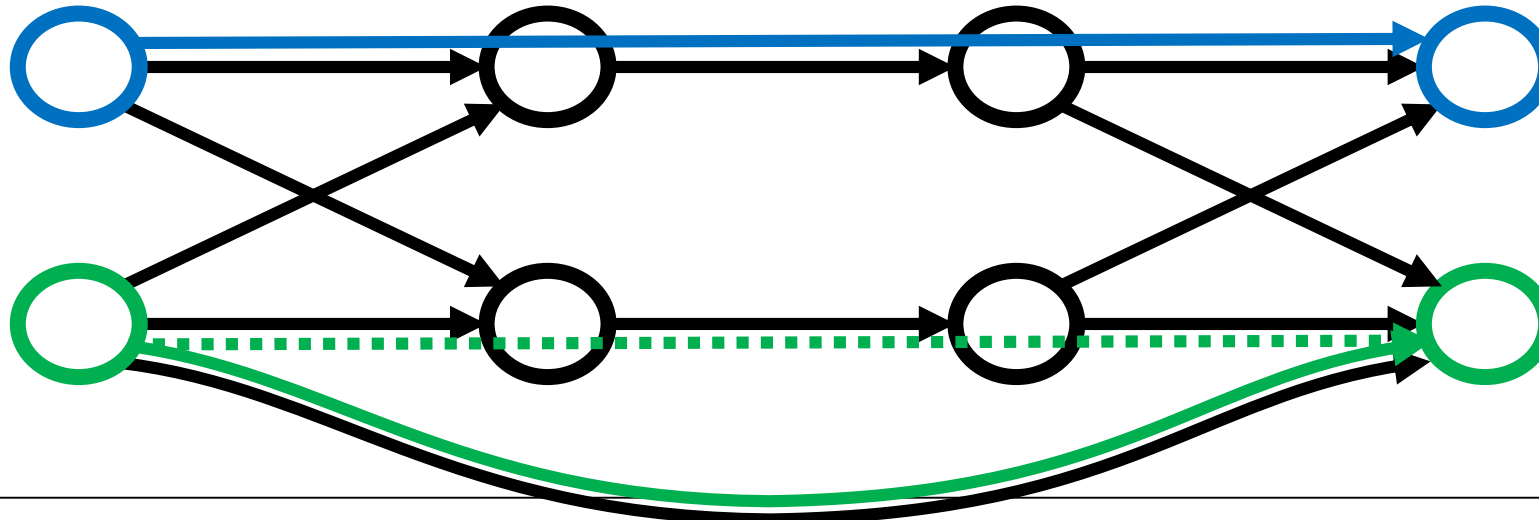
1.  $x := b$ , 2.  $b := g$ , 3.  $g := x$



## Consistent Migration of Splittable Flows

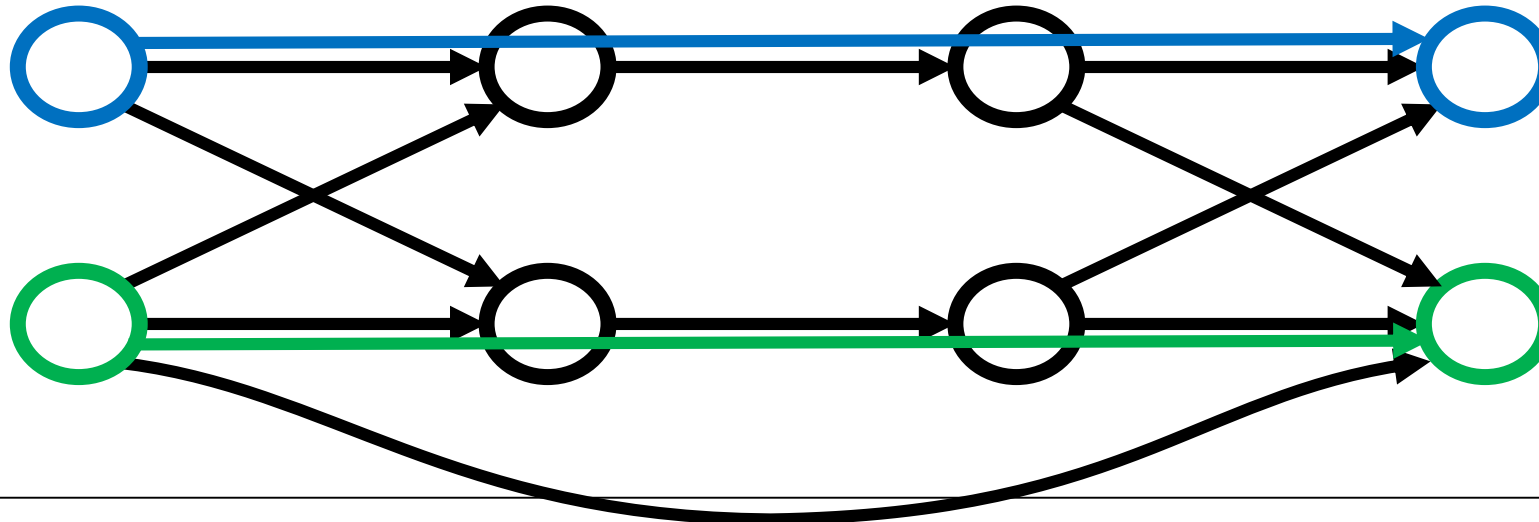
Think: variable swapping of  $b$  &  $g$

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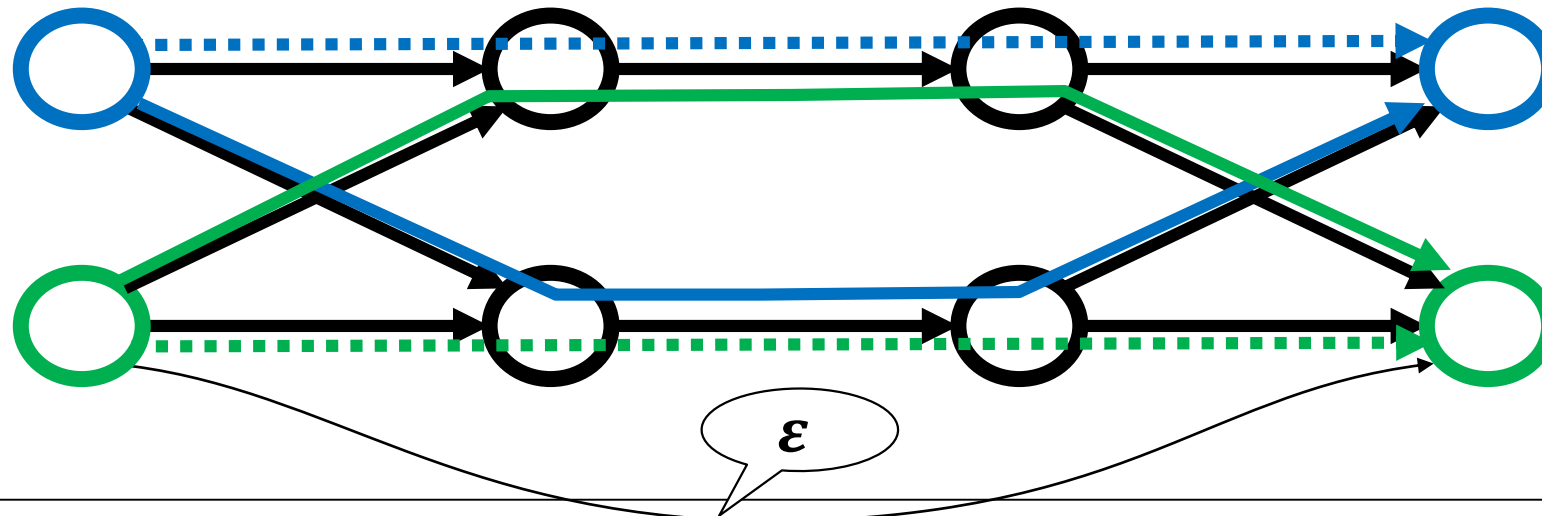
## Consistent Migration of Splittable Flows

SWAN: LP-approach with binary search  
1 update? 2 updates? 4 updates? ...



## Consistent Migration of Splittable Flows

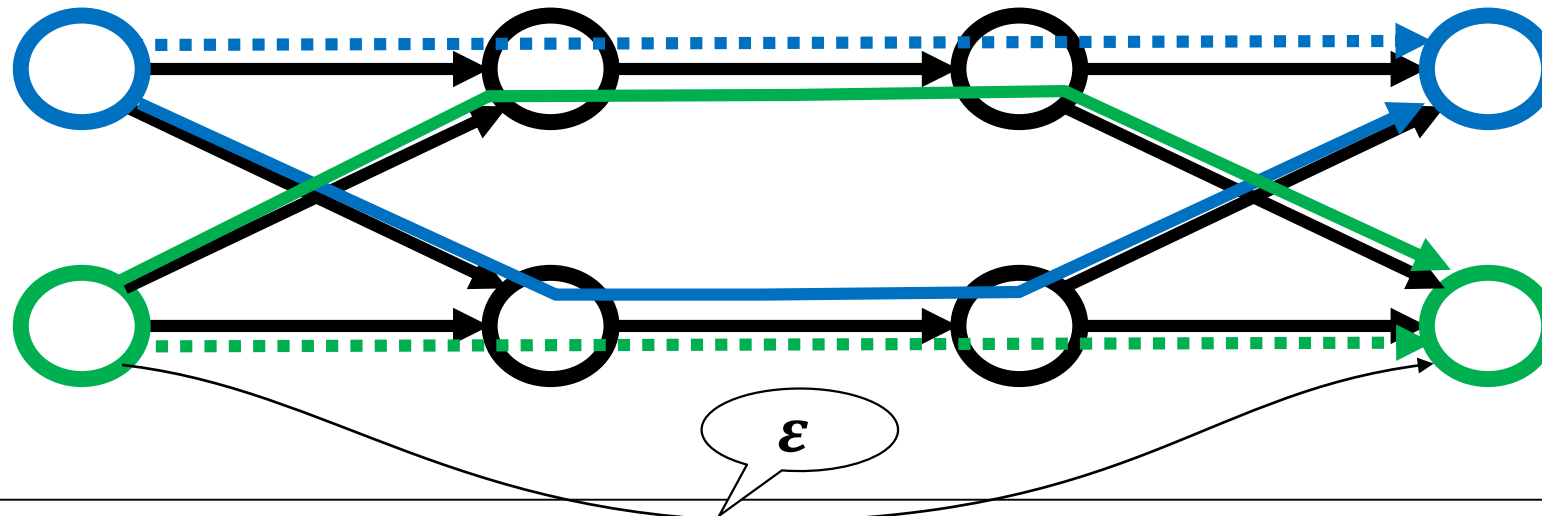
SWAN: LP-approach with binary search  
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## Consistent Migration of Splittable Flows

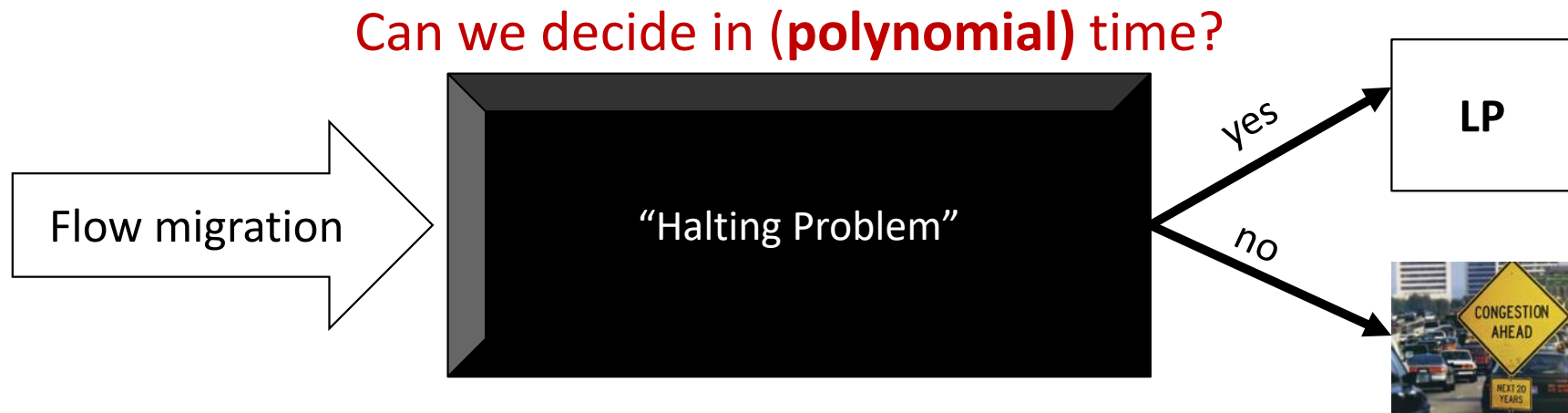
SWAN: LP-approach with binary search

$\Theta(1/\varepsilon)$  updates ☹️





## Consistent Migration of Splittable Flows



## To Slack or not to Slack?

Slack of  $x$  on all flow edges?  
 $\lceil 1/x \rceil - 1$  updates

## To Slack or not to Slack?

What if not?

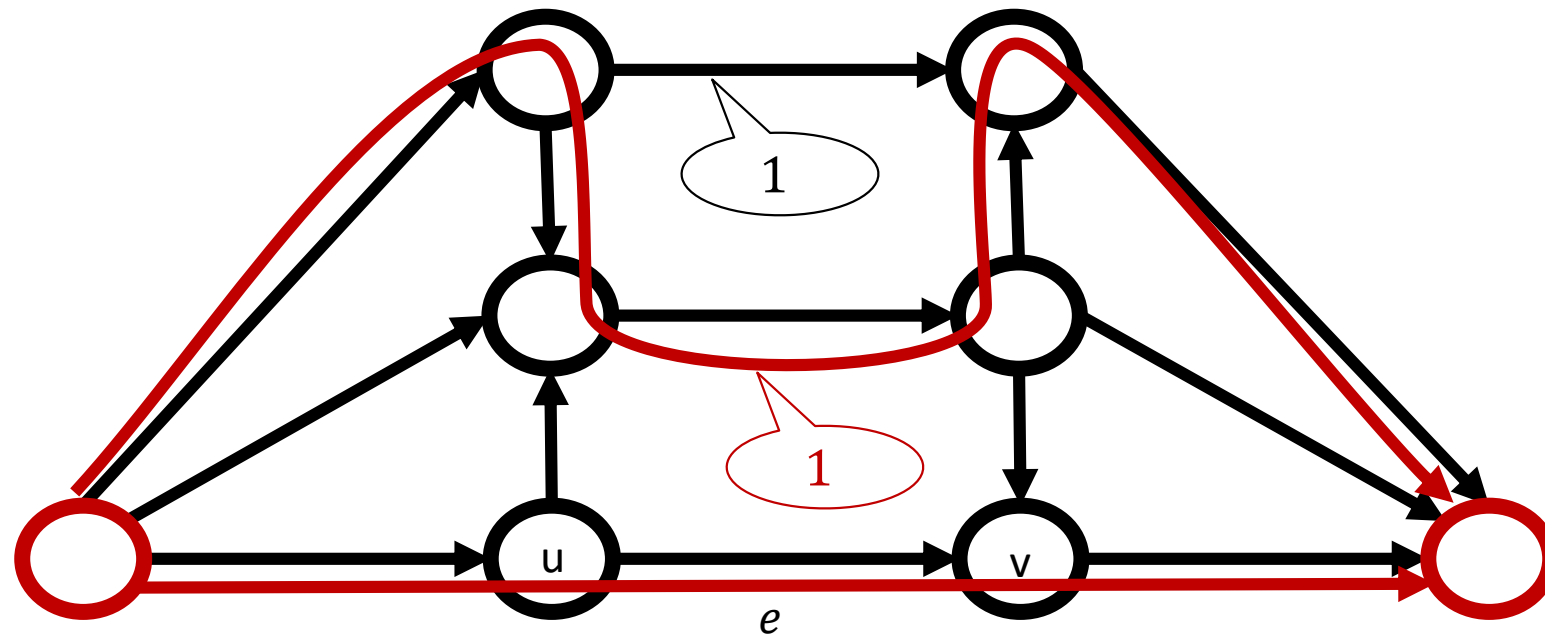
Try to create slack

## To Slack or not to Slack?

Combinatorial approach  
Augmenting paths

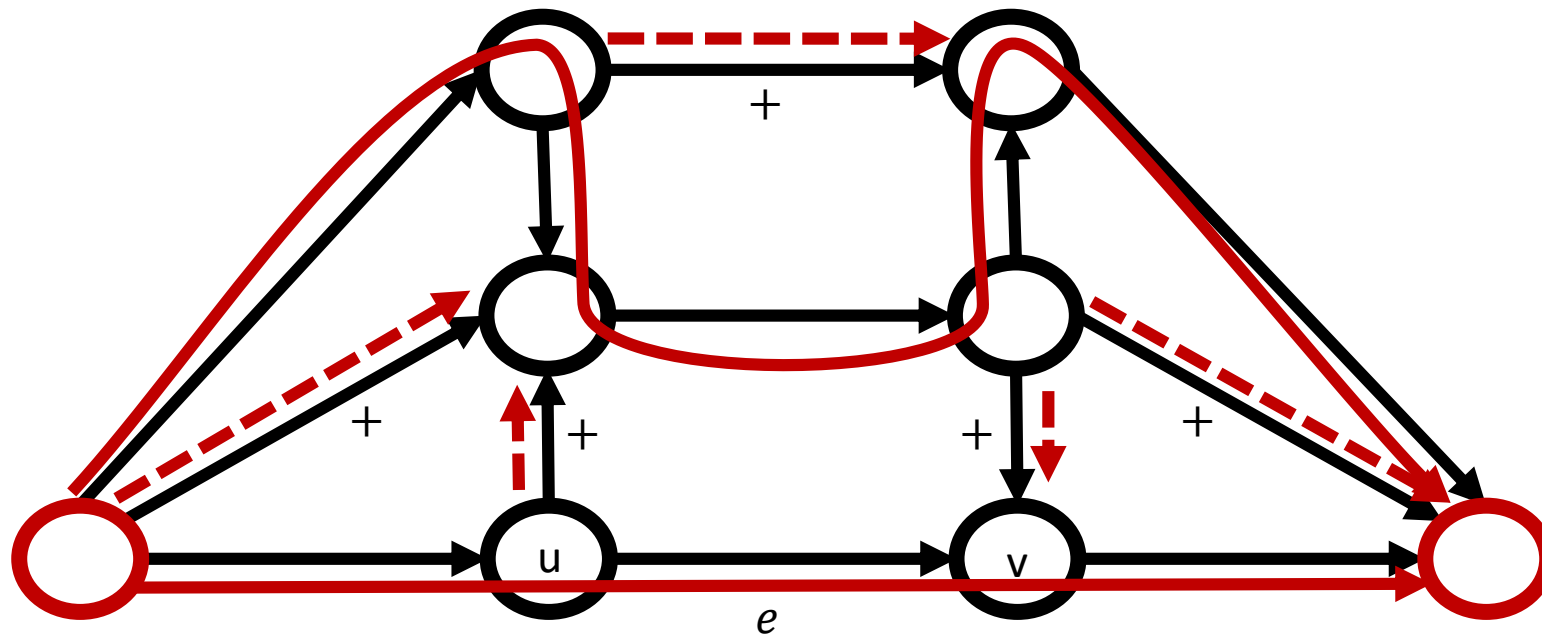
## Combinatorial Approach

Move single commodities at a time



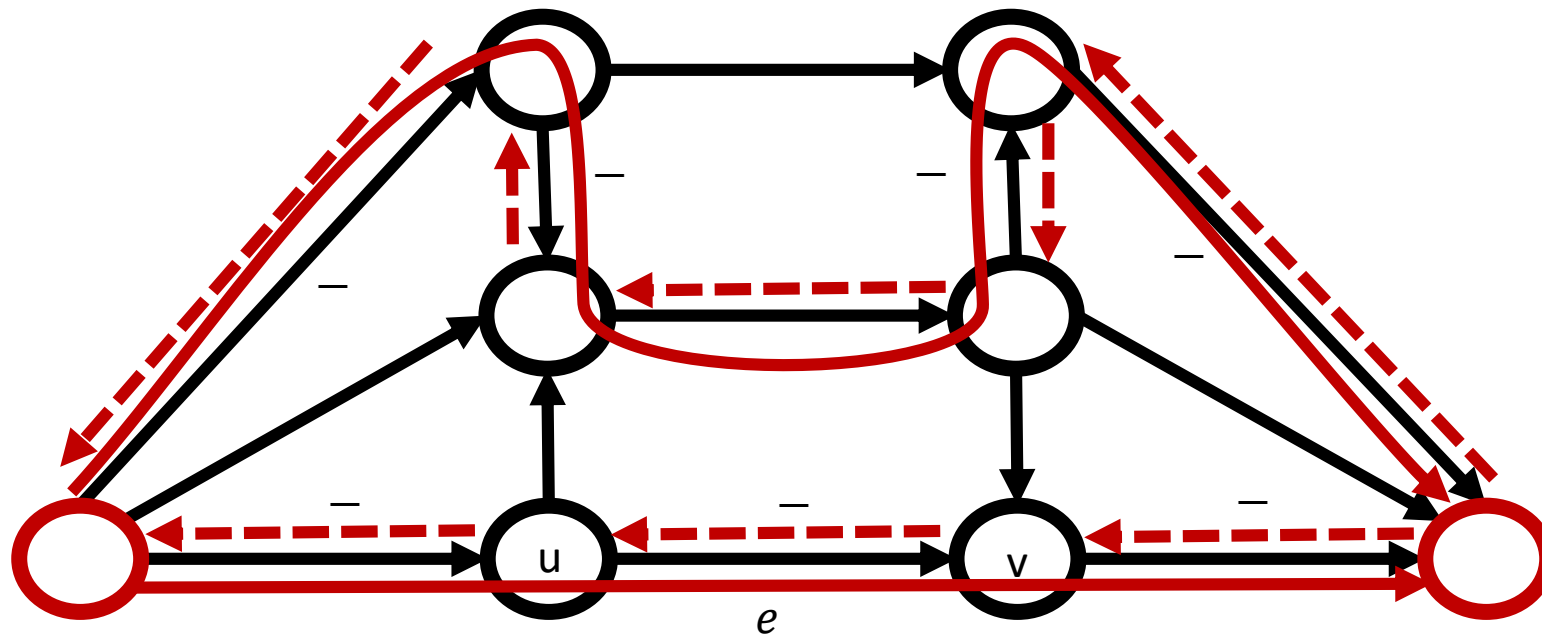
## Combinatorial Approach

Where to increase flow?



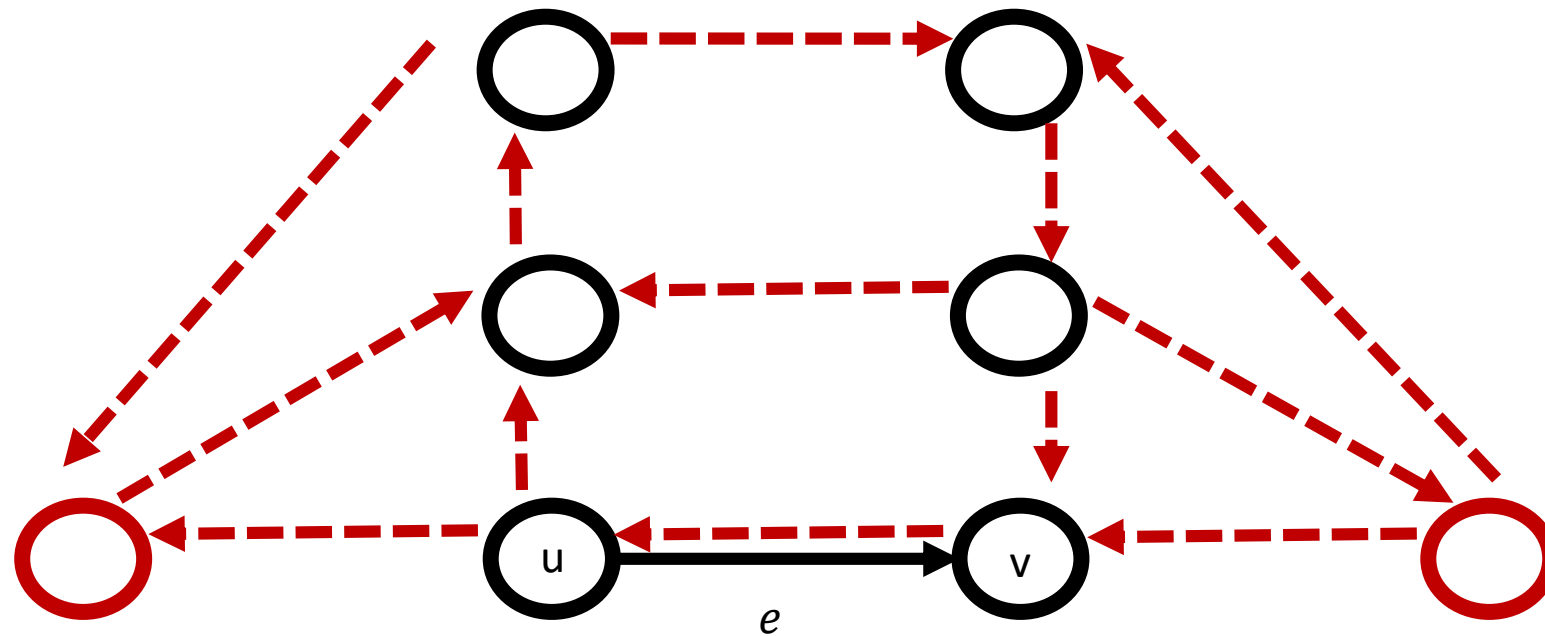
## Combinatorial Approach

Where to push back flow?



## Combinatorial Approach

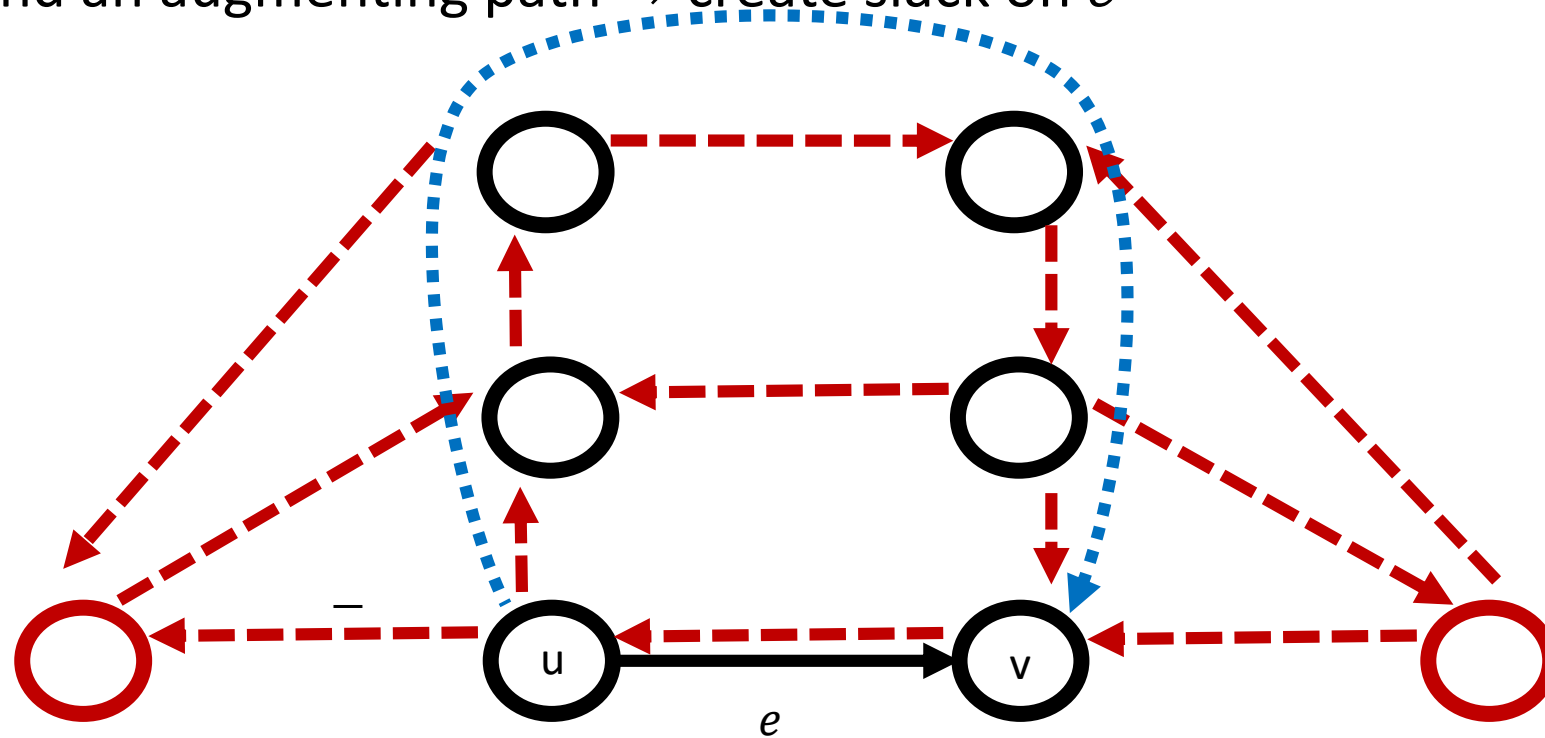
Resulting residual network





## Combinatorial Approach

We found an augmenting path  $\Rightarrow$  create slack on  $e$



## High-level Algorithm Idea

- No slack on flow edges? Find augmenting paths
  - On both initial and desired state (updates can be performed in reverse)
  - Success? Use *SWAN* method to migrate
  
- Can't create slack on some flow edge?
  - Consistent migration impossible  
By contradiction (else augmenting paths would create slack)
  
- Runtime:  $O(Fm^3)$ 
  - ( $F$  being #commodities,  $m$  being #edges)

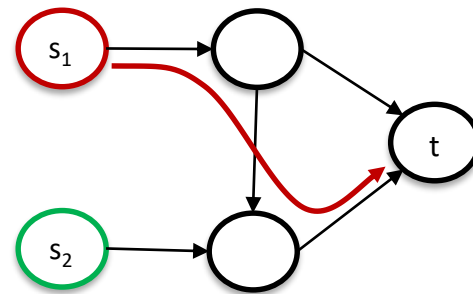
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*On Consistent Migration of Flows in SDNs.* S. Brandt, K.-T. Foerster, R. Wattenhofer, INFOCOM 2016

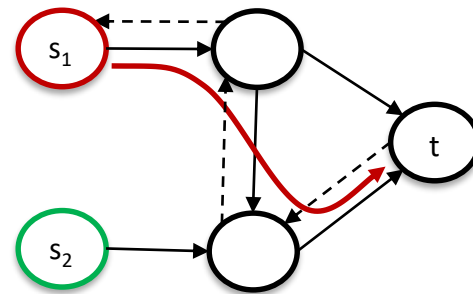
Maybe surprisingly:  
If the new flows fit in somehow,  
we can migrate consistently!

## Open problems for scheduling flow migration

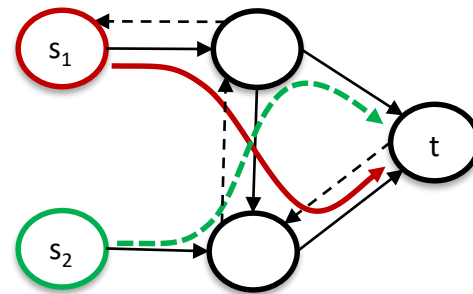
- What happens when we can pick the new paths?
  - Idea: Fit the flows in, does not matter where
    - Only studied so far for a single destination and multiple sources [Brand, Foerster, Wattenhofer, PMC 2017]



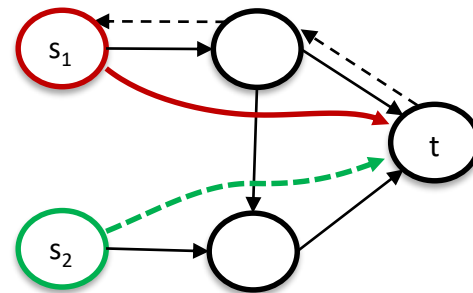
size of each flow: 1  
capacity of links: 1



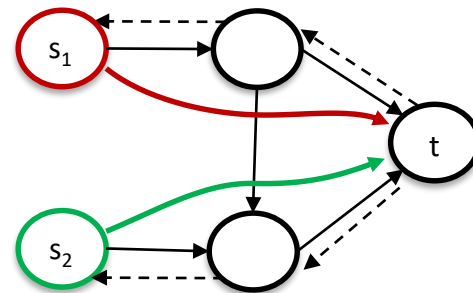
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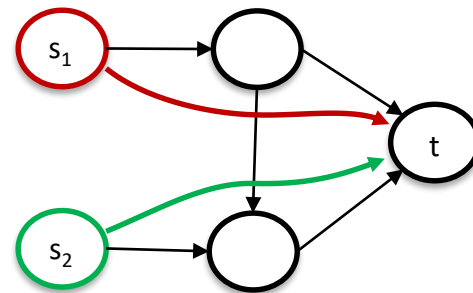


size of each flow: 1  
capacity of links: 1

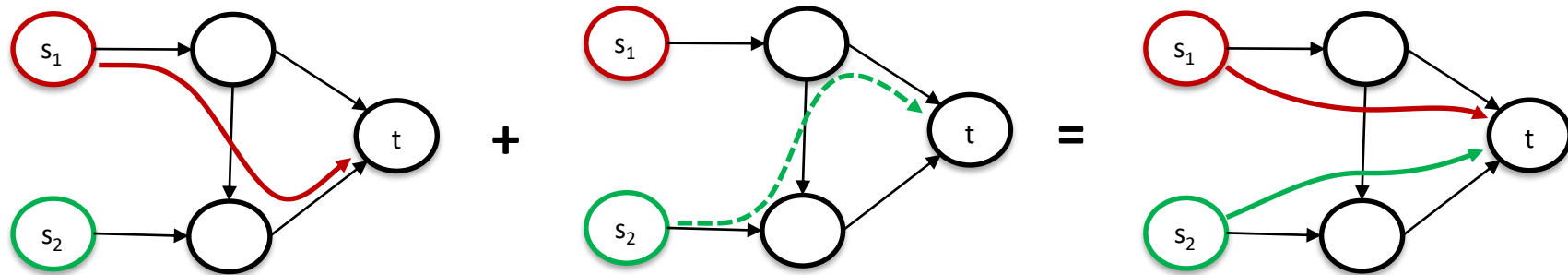


size of each flow: 1  
capacity of links: 1

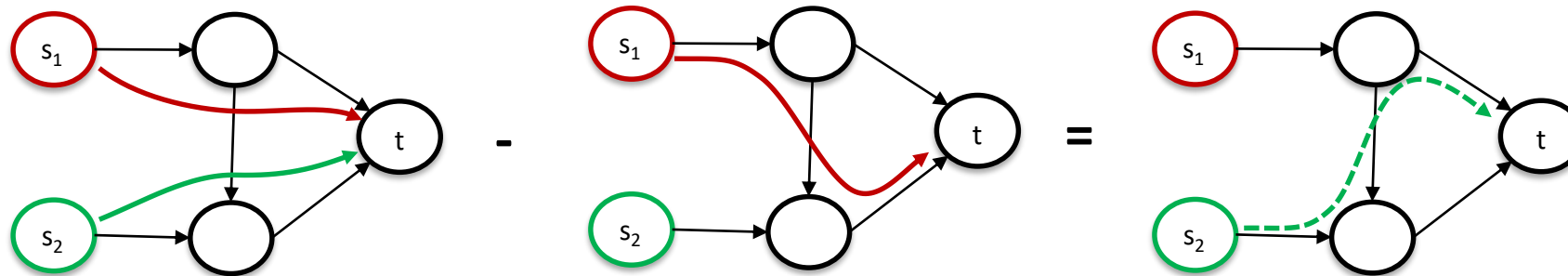




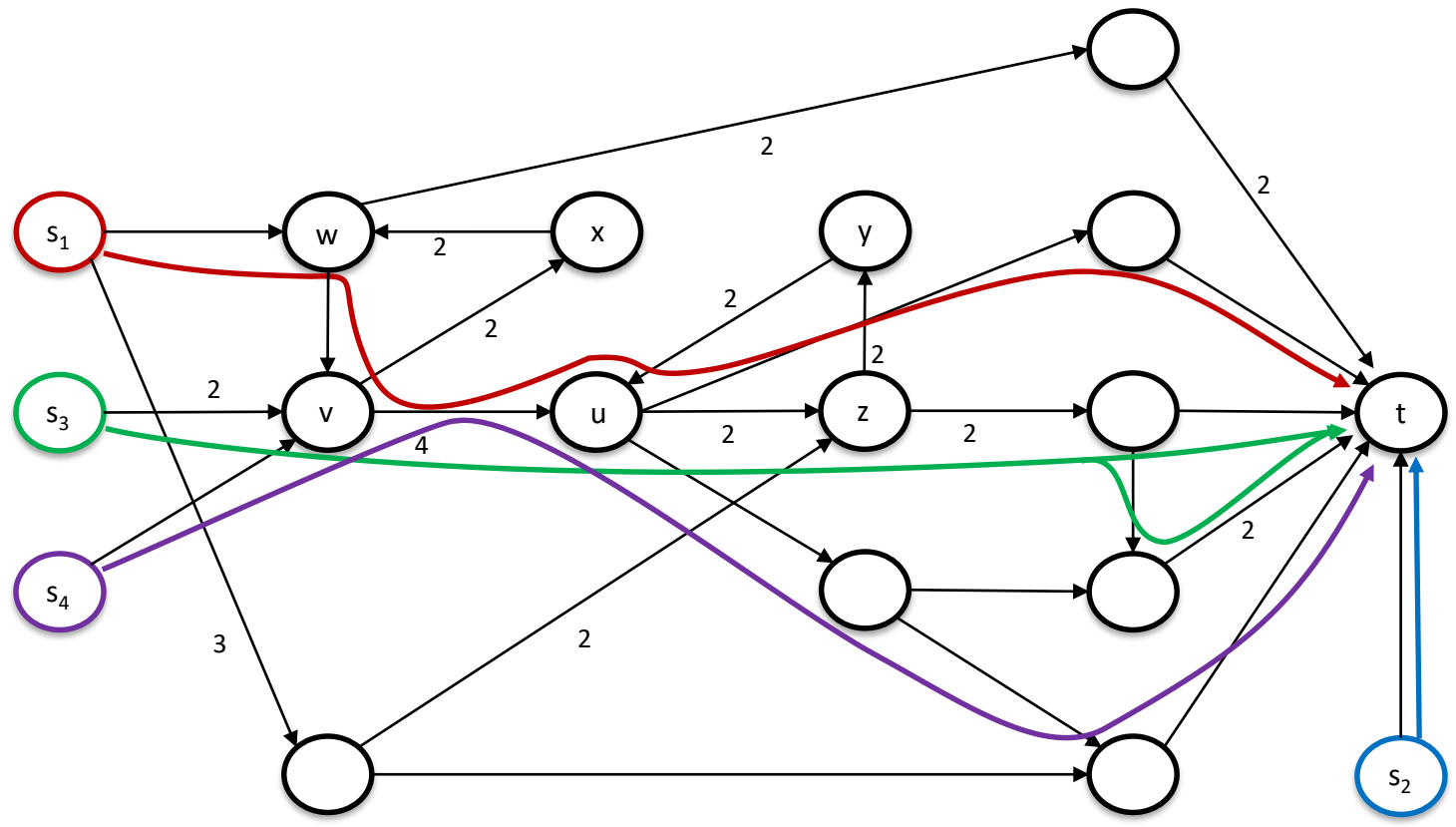
size of each flow: 1  
capacity of links: 1

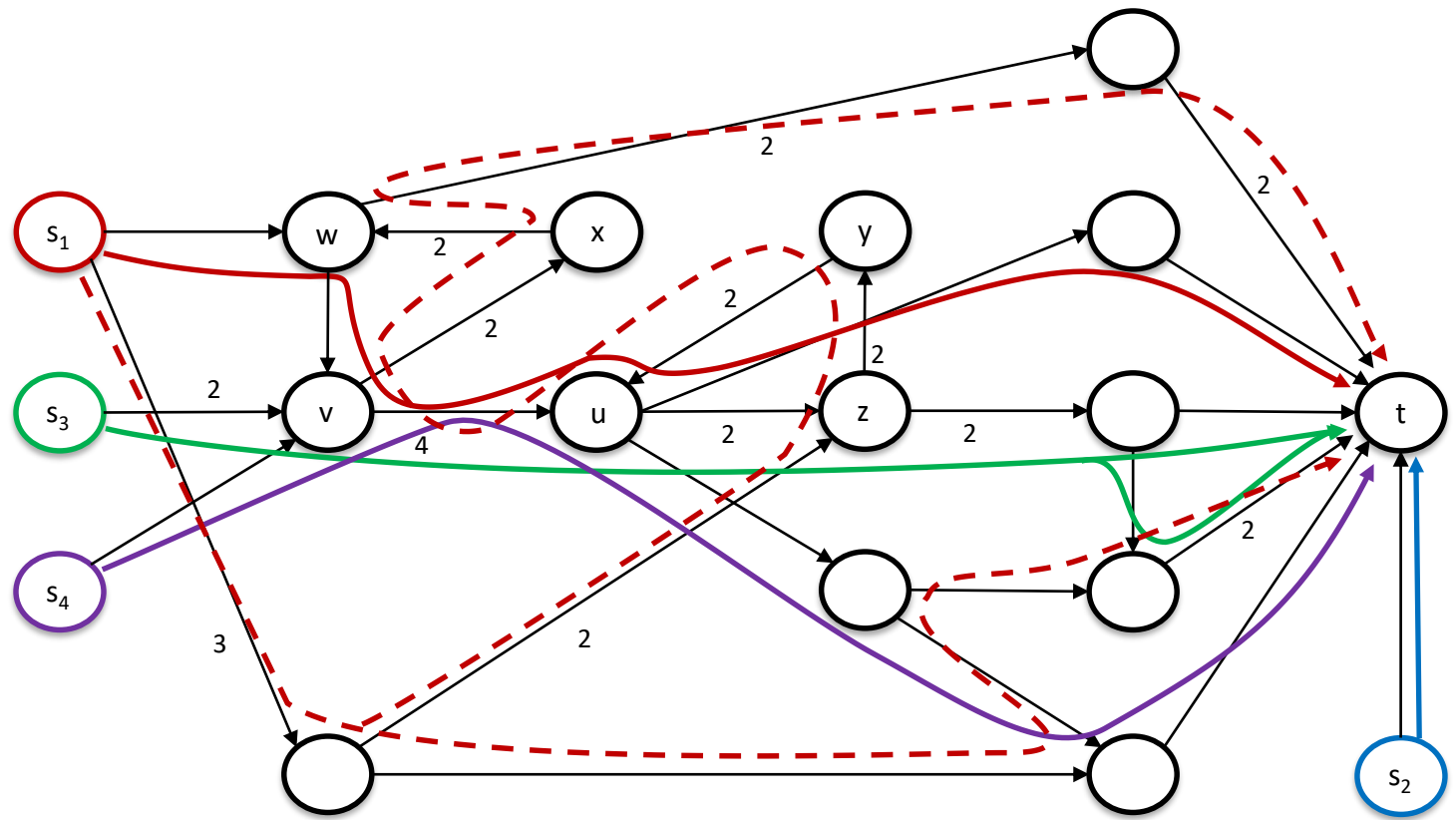


size of each flow: 1  
 capacity of links: 1

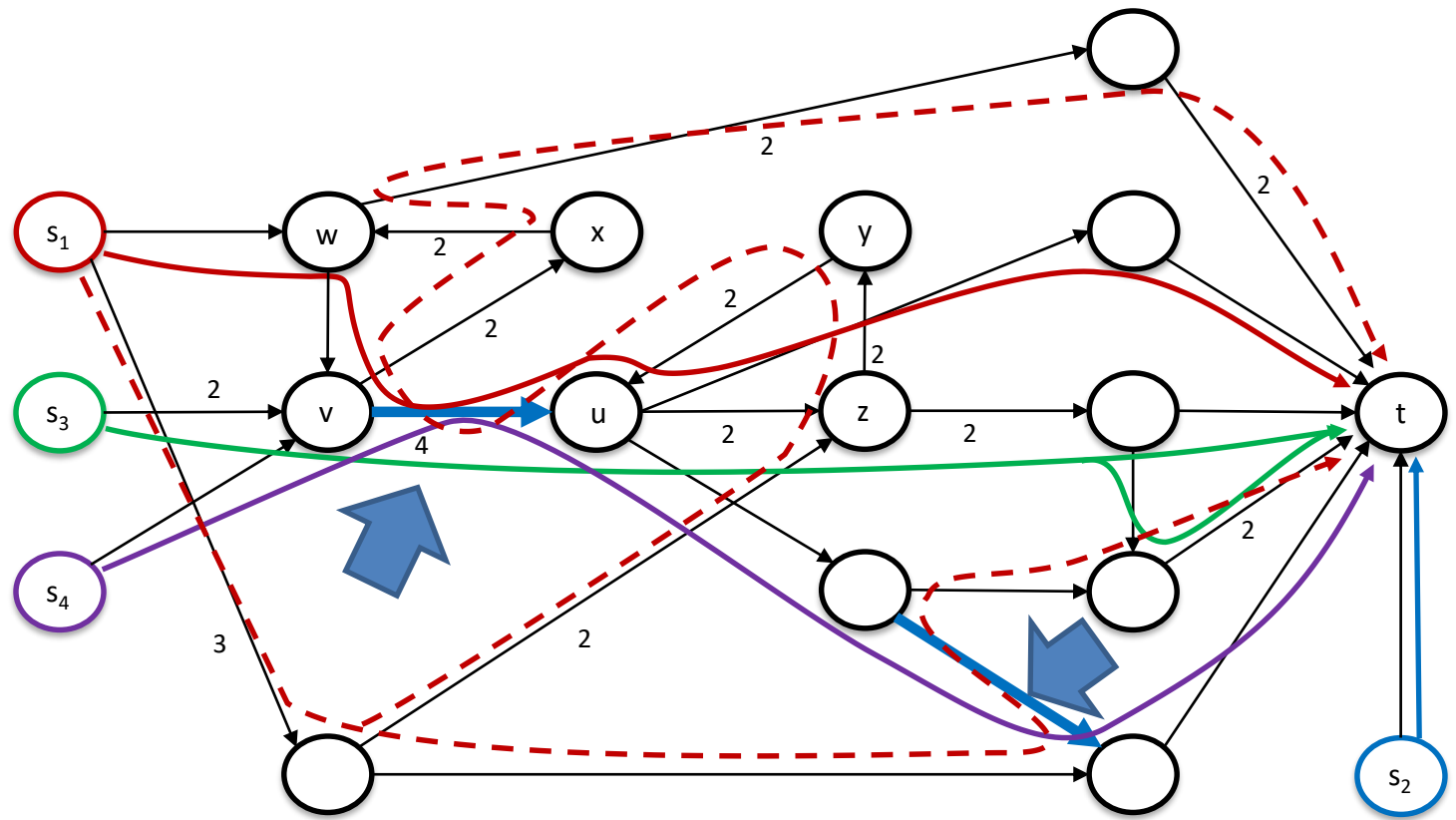


size of each flow: 1  
 capacity of links: 1

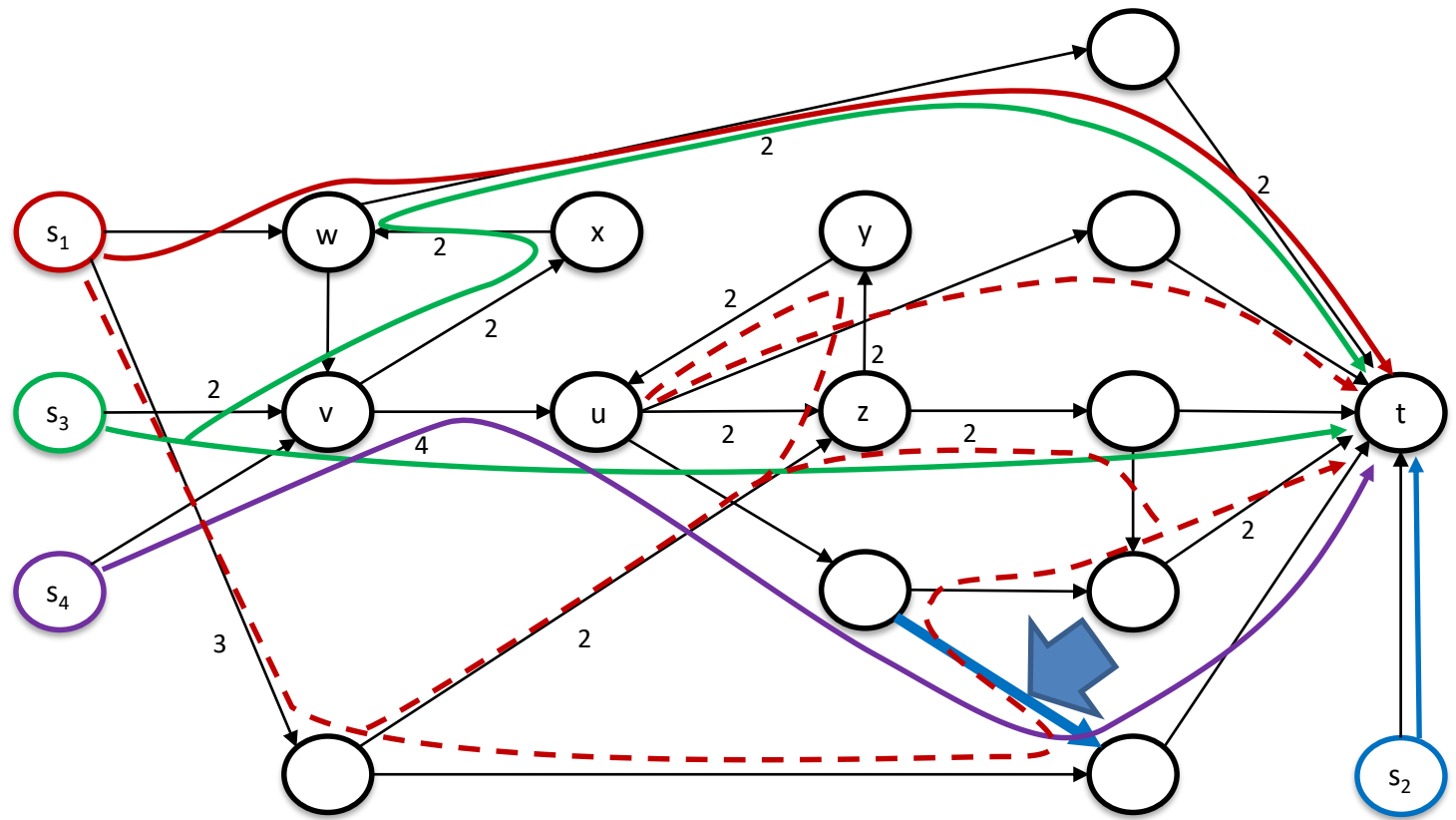




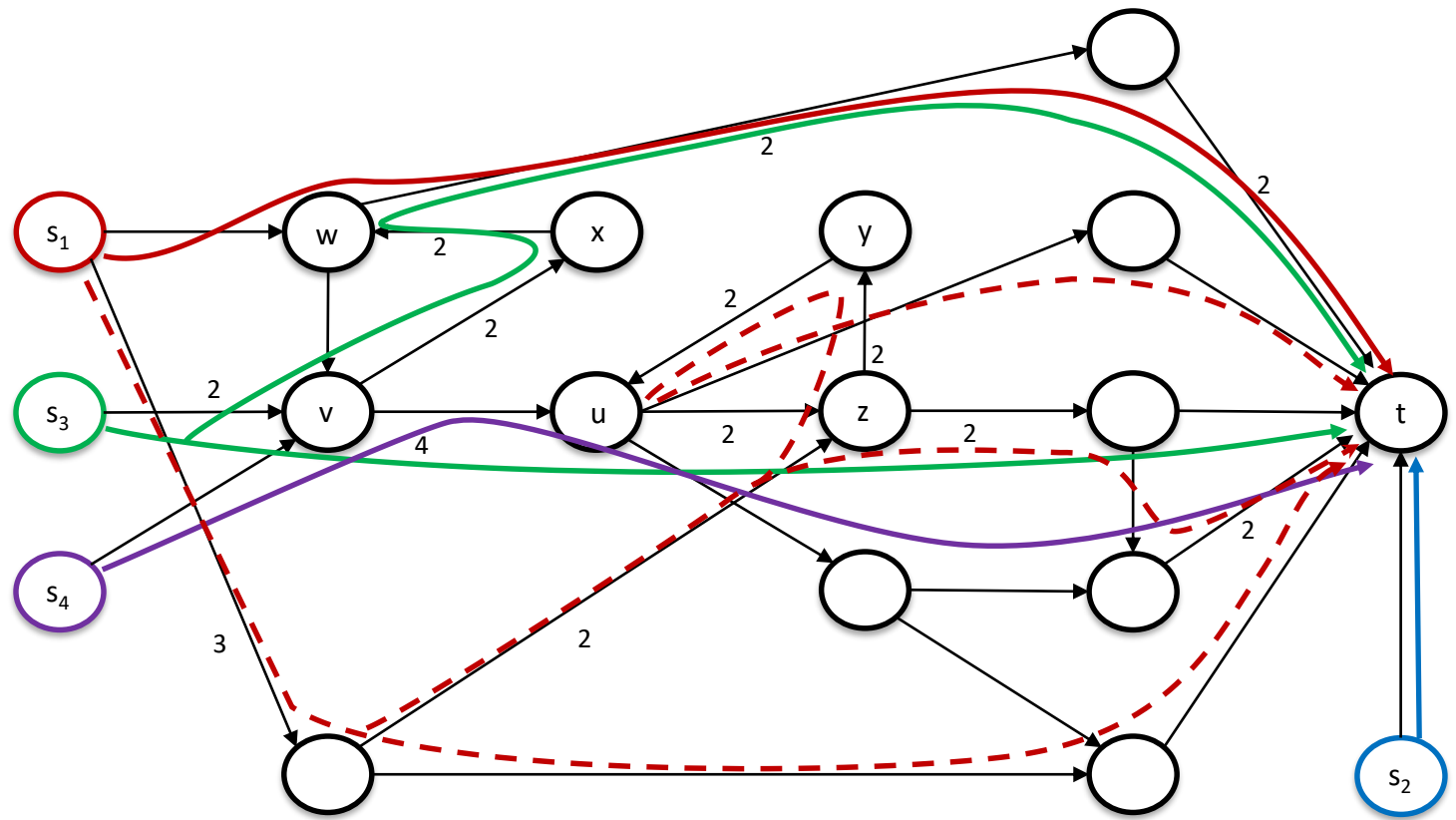
size of flows: 1, 2, 1, 1  
 capacity of links: 1 (or marked)



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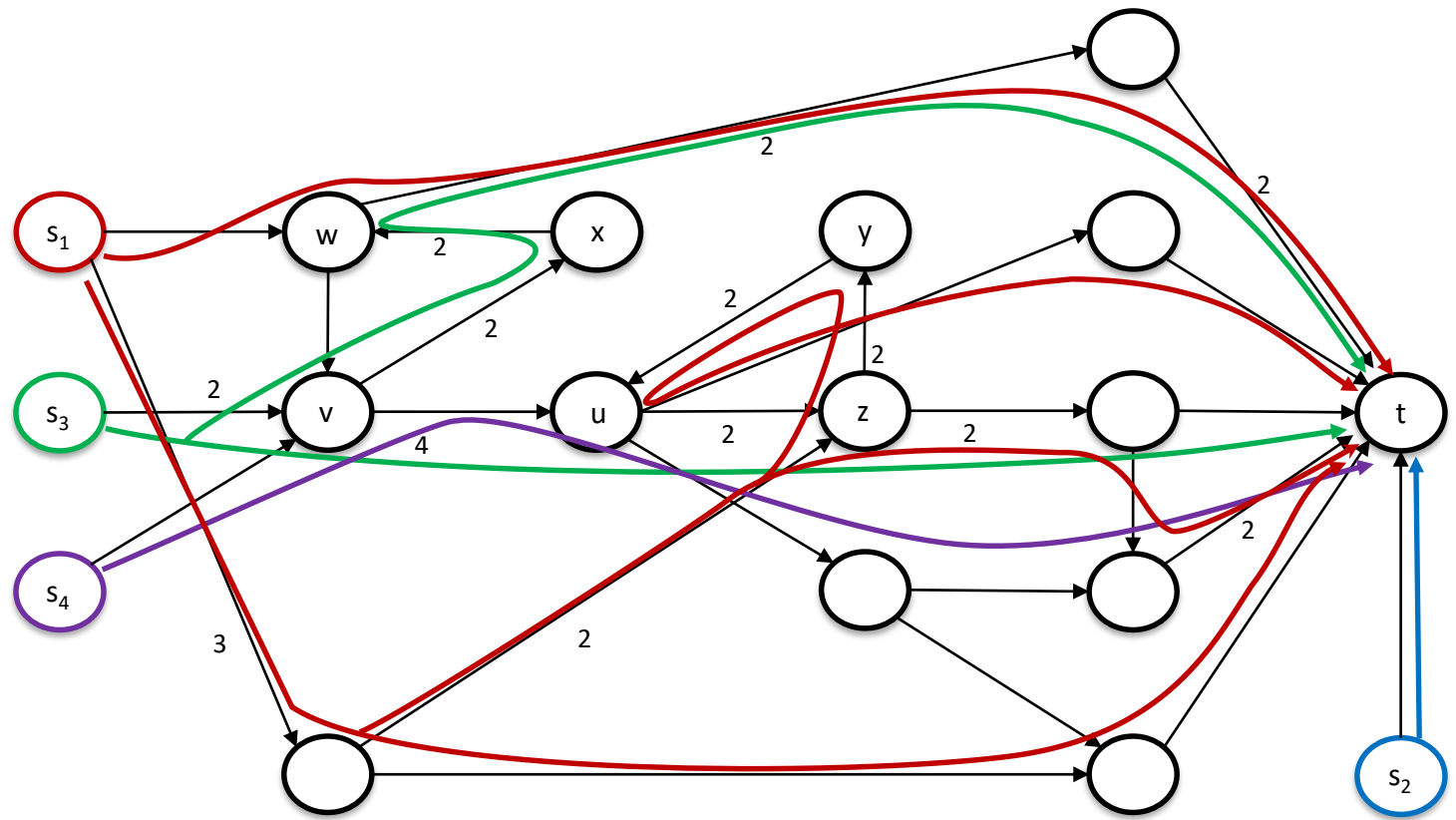


size of flows: 1, 2, 1, 1  
 capacity of links: 1 (or marked)

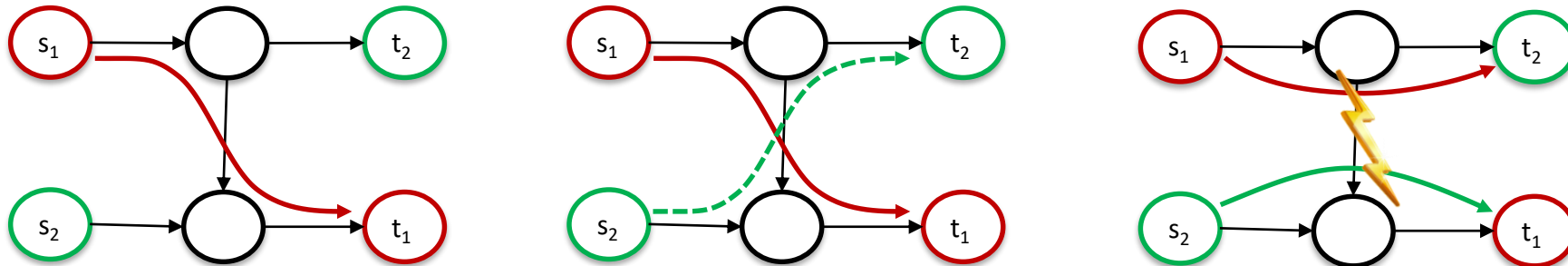


size of flows: 1, 2, 1, 1  
 capacity of links: 1 (or marked)

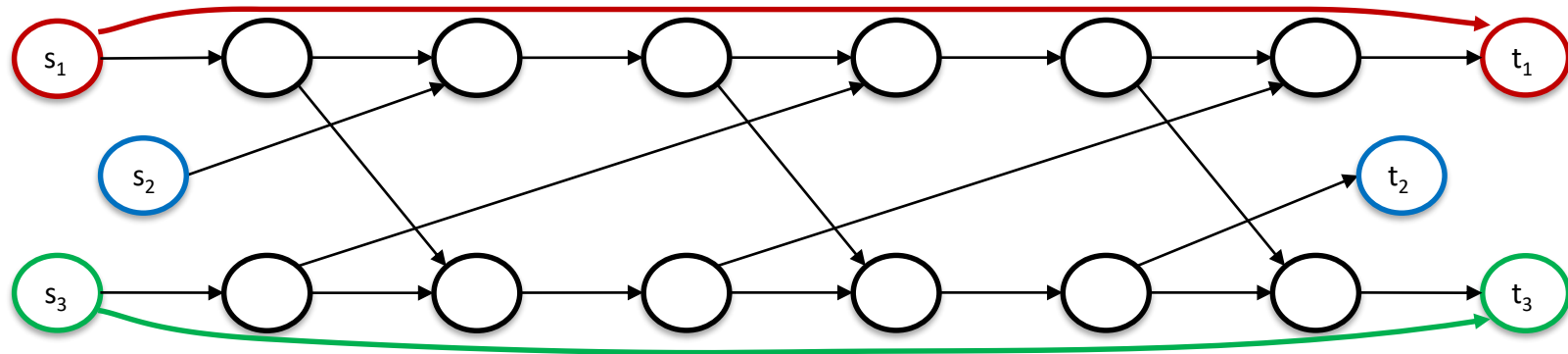




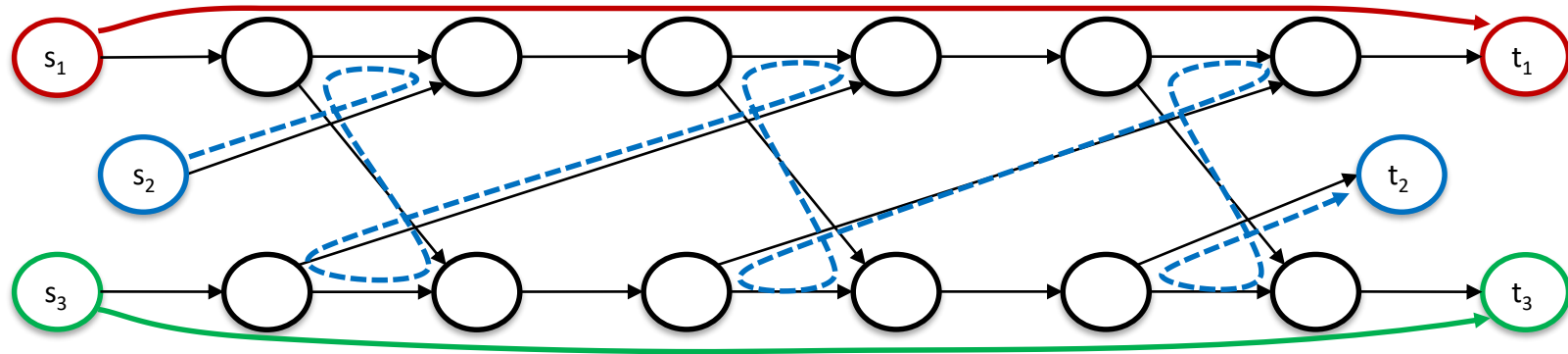
size of flows:  $1+3=4$ , 2, 1, 1  
 capacity of links: 1 (or marked)



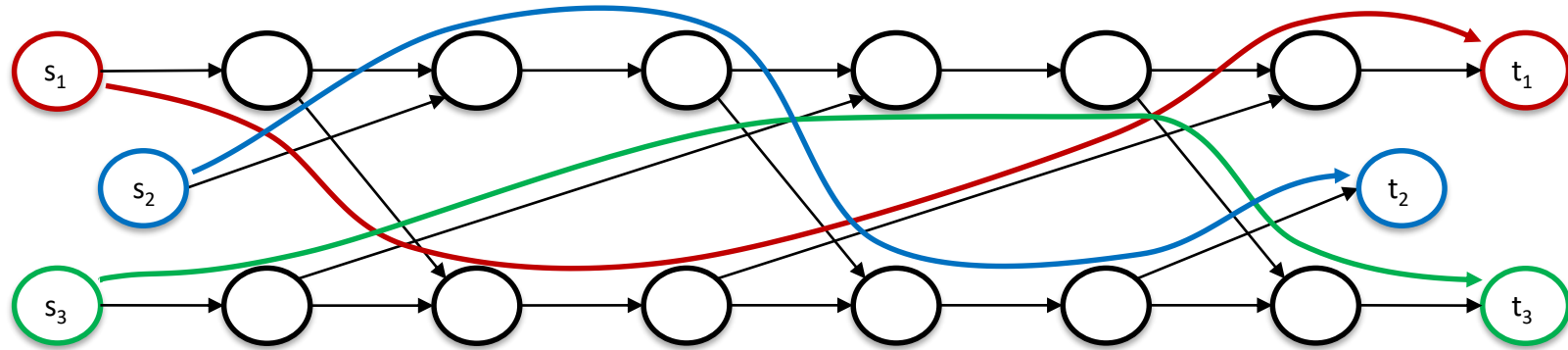
- Flows end up at the wrong destination!
- So let's stick with augmenting flows that don't mix destinations



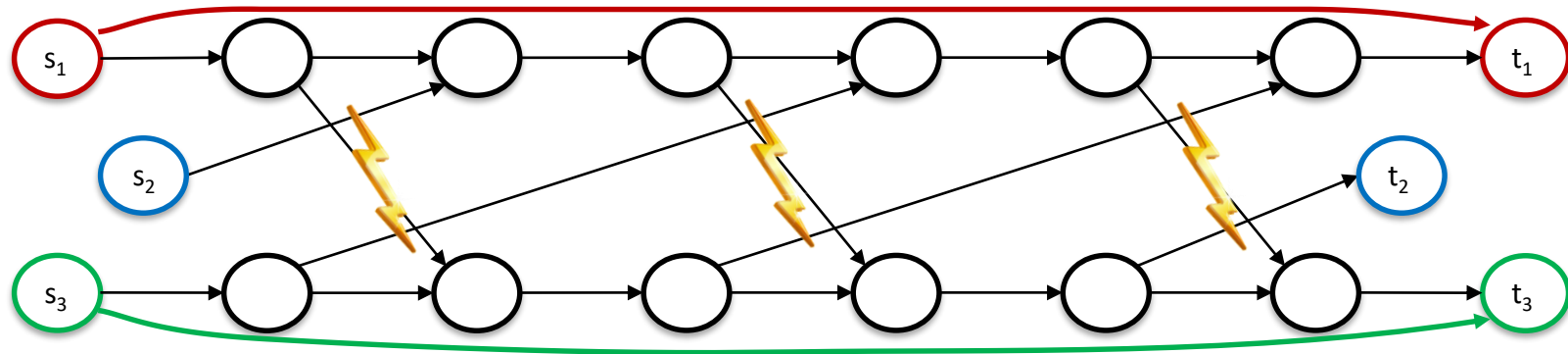
size of each flow: 1  
 capacity of each links: 1



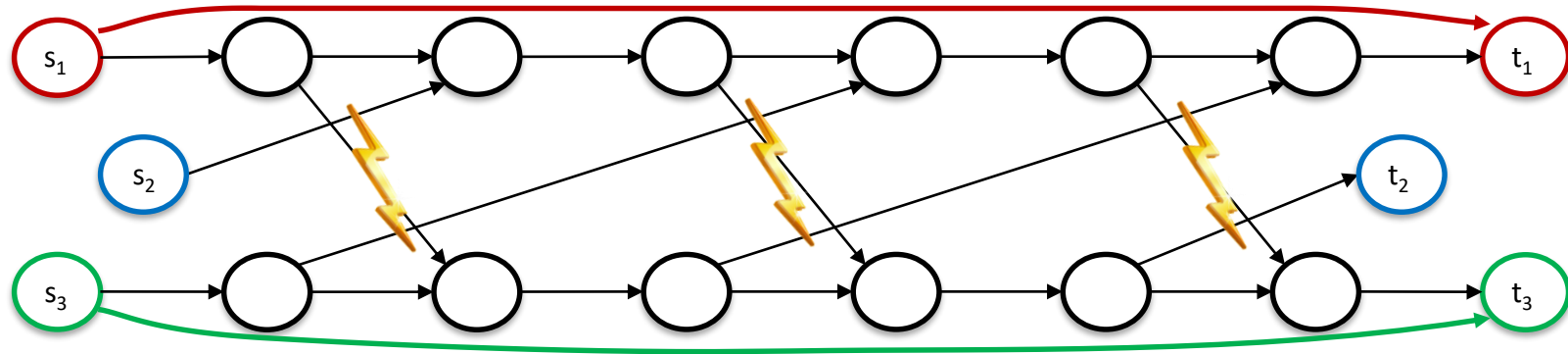
size of each flow: 1  
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size of each flow: 1  
 capacity of each links: 1



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 capacity of each links: 1



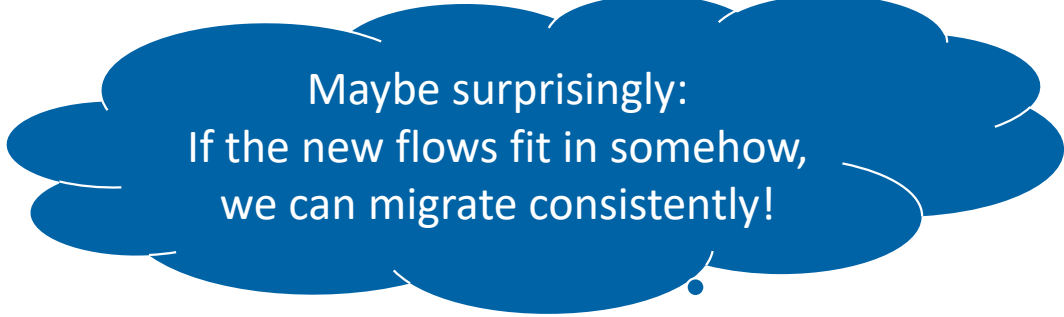
*“it is unlikely that similar techniques can be developed  
for constructing multicommodity flows”*

[Hu, 1963]

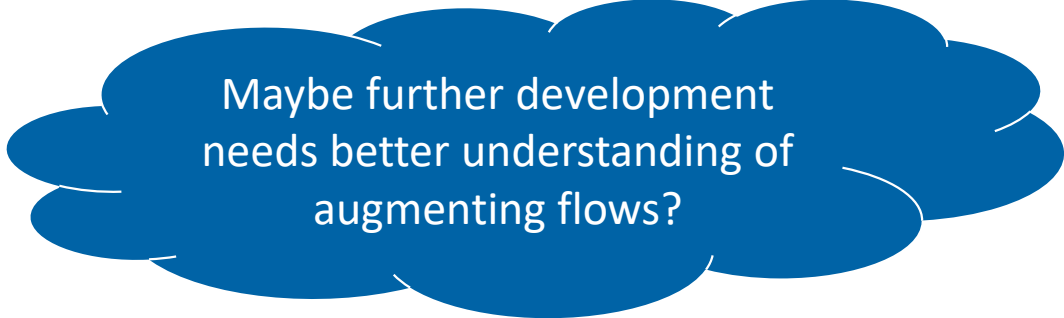
size of each flow: 1  
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## Open Problems for scheduling flow migration

- What happens when we can pick the new paths?
  - Idea: Fit the flows in, does not matter where
    - Only studied so far for a single destination and multiple sources [Brand, Foerster, Wattenhofer, PMC 2017]
- Unsplittable flow migration:
  - In general: NP-, PSPACE-, or EXPTIME-complete?
    - (recall: flows might need to switch back and forth repeatedly)
  - "Interesting" polynomial cases?



Maybe surprisingly:  
If the new flows fit in somehow,  
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Maybe further development  
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## Open Problems for scheduling flow migration

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- Unsplittable flow migration:
  - In general: NP-, PSPACE-, or EXPTIME-complete?
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  - "Interesting" polynomial cases?
- What happens when considering **Link Latency**?

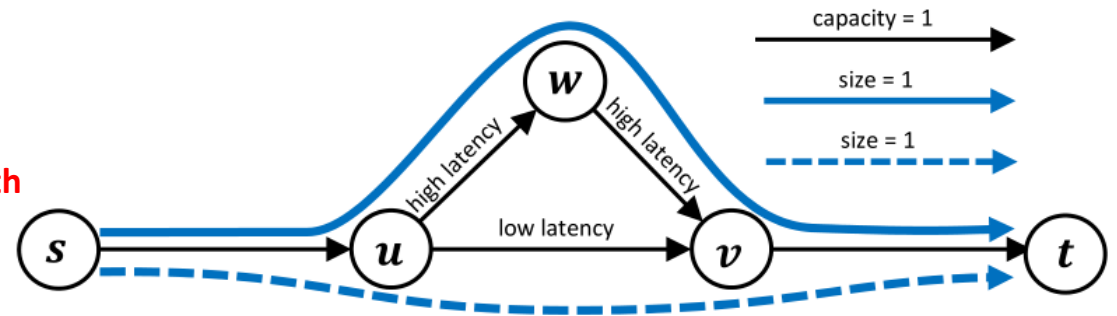
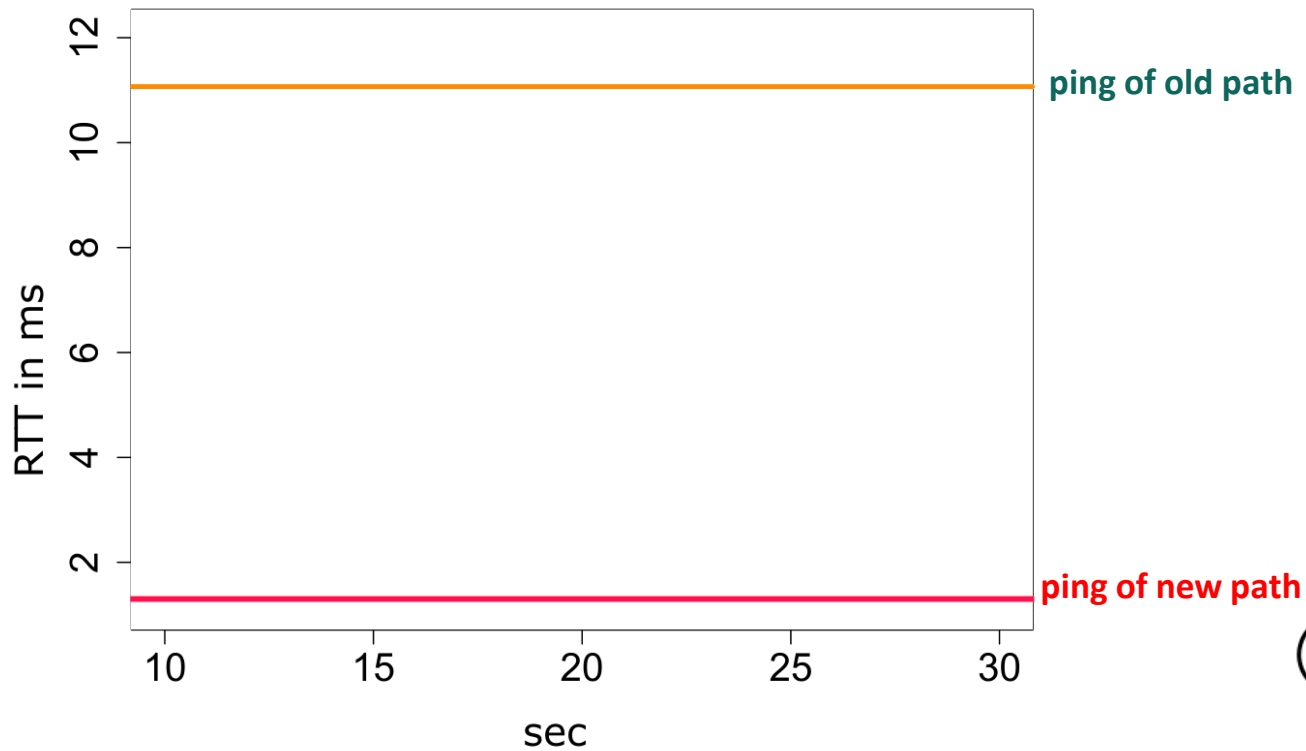
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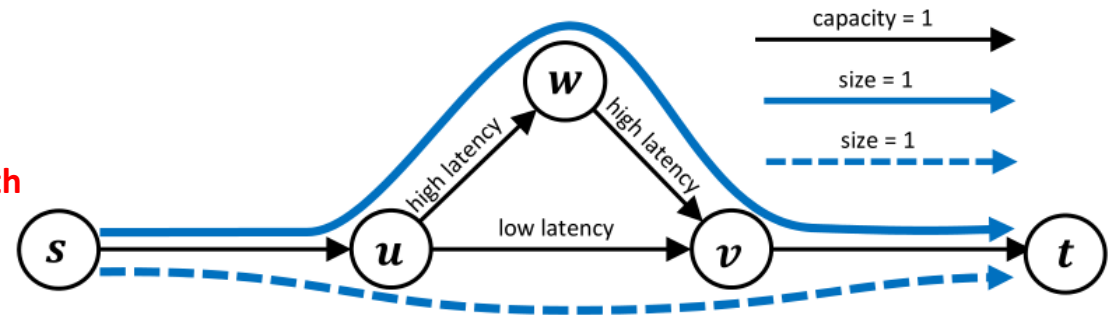
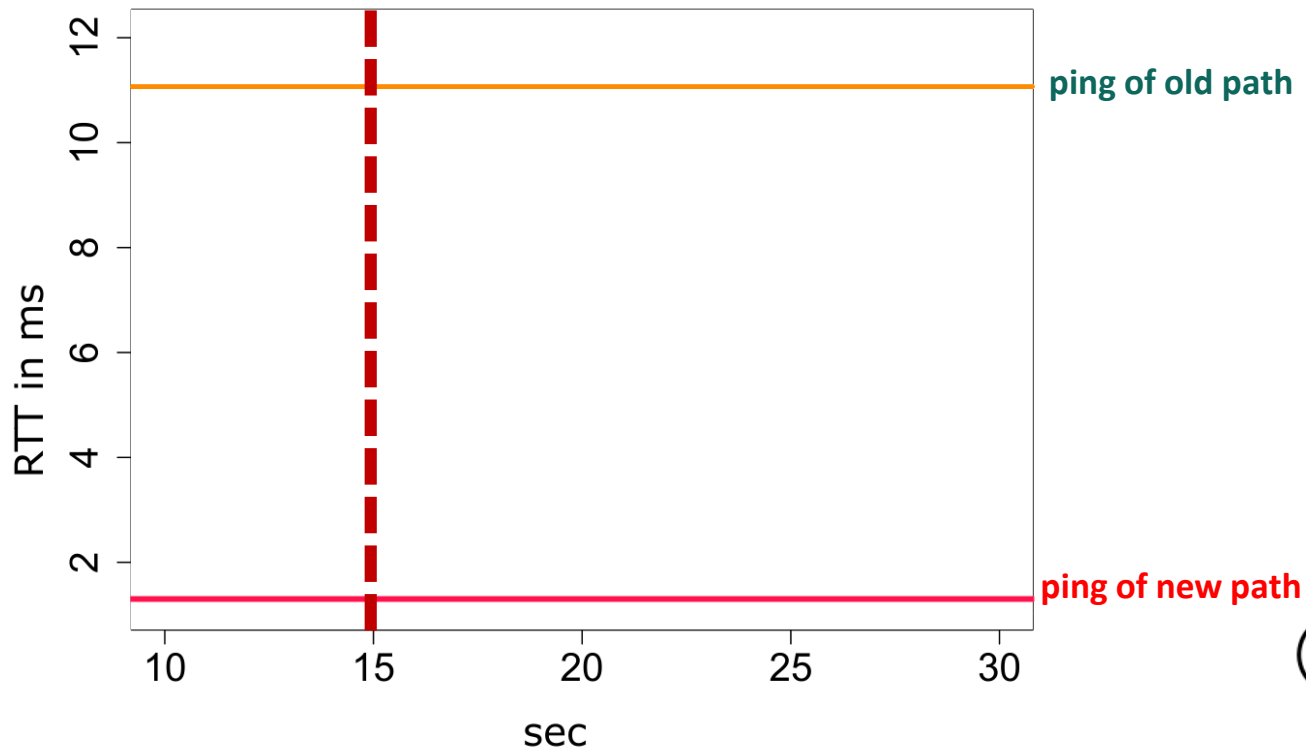
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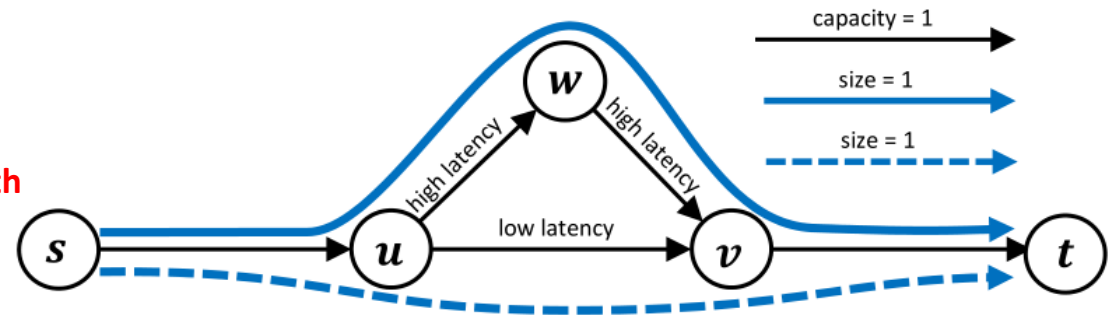
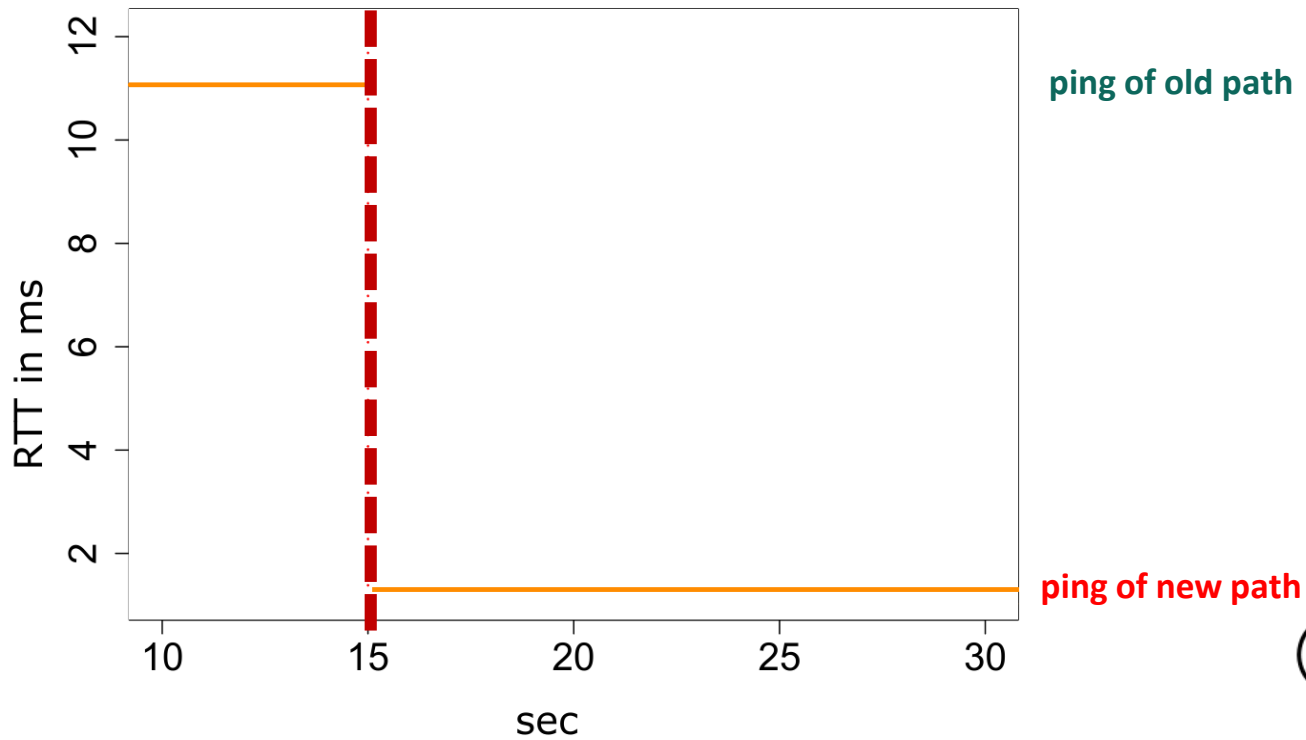
## The Impact of Latency (in Testbed)



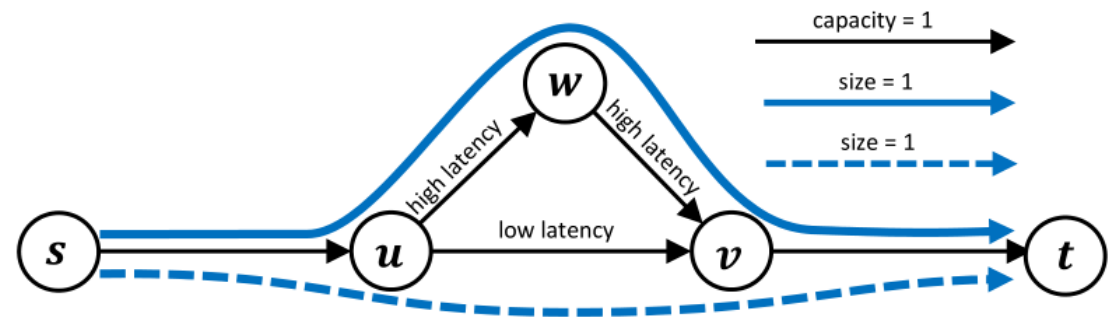
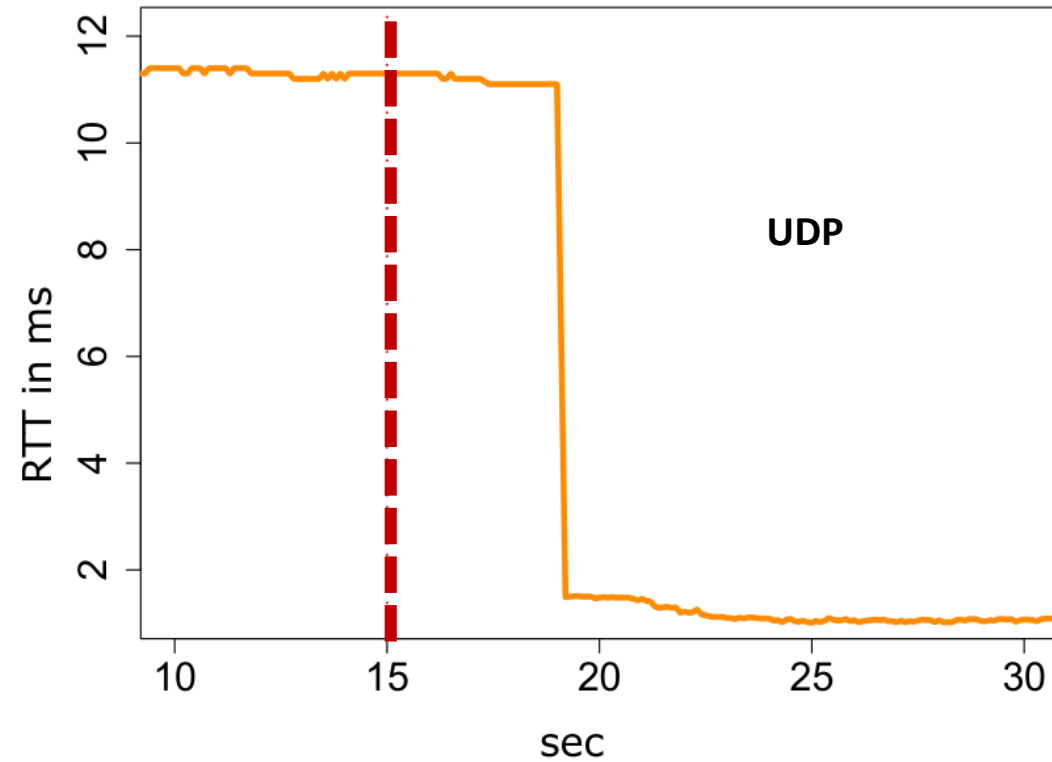
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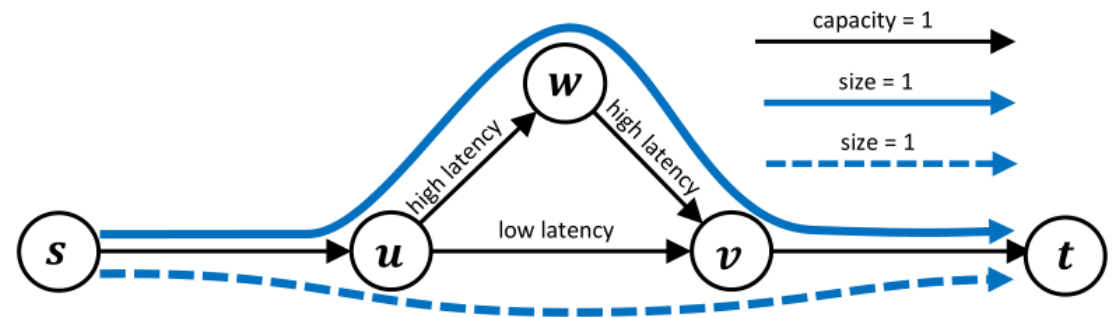
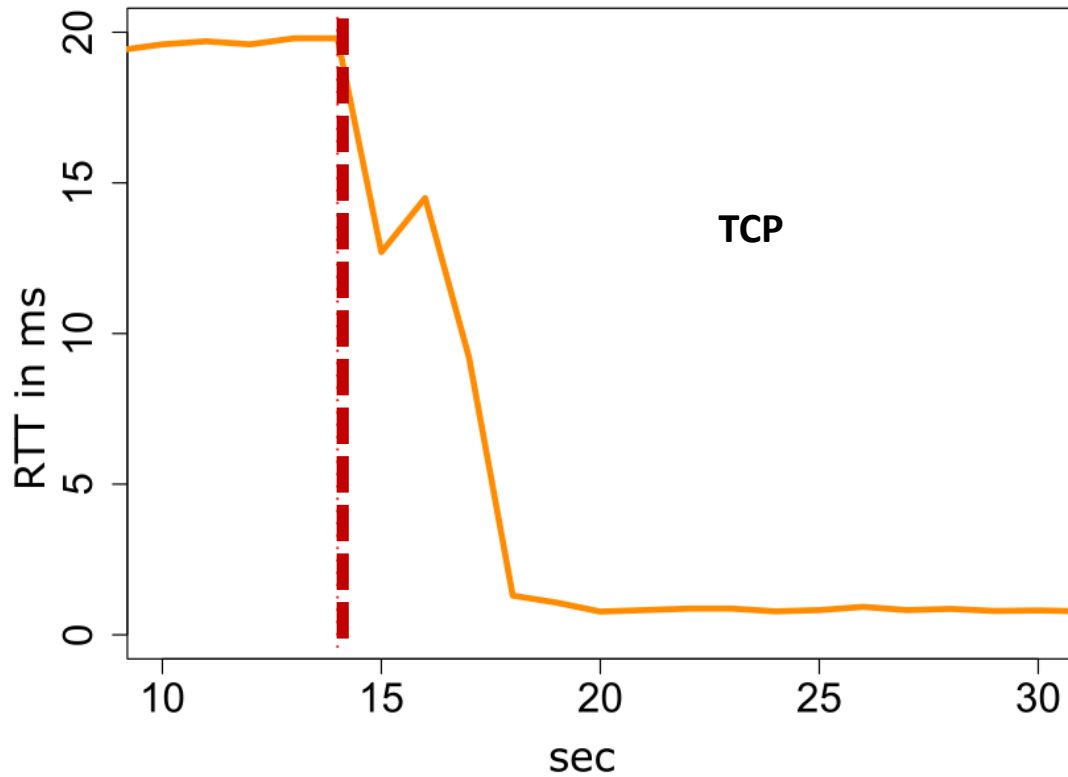
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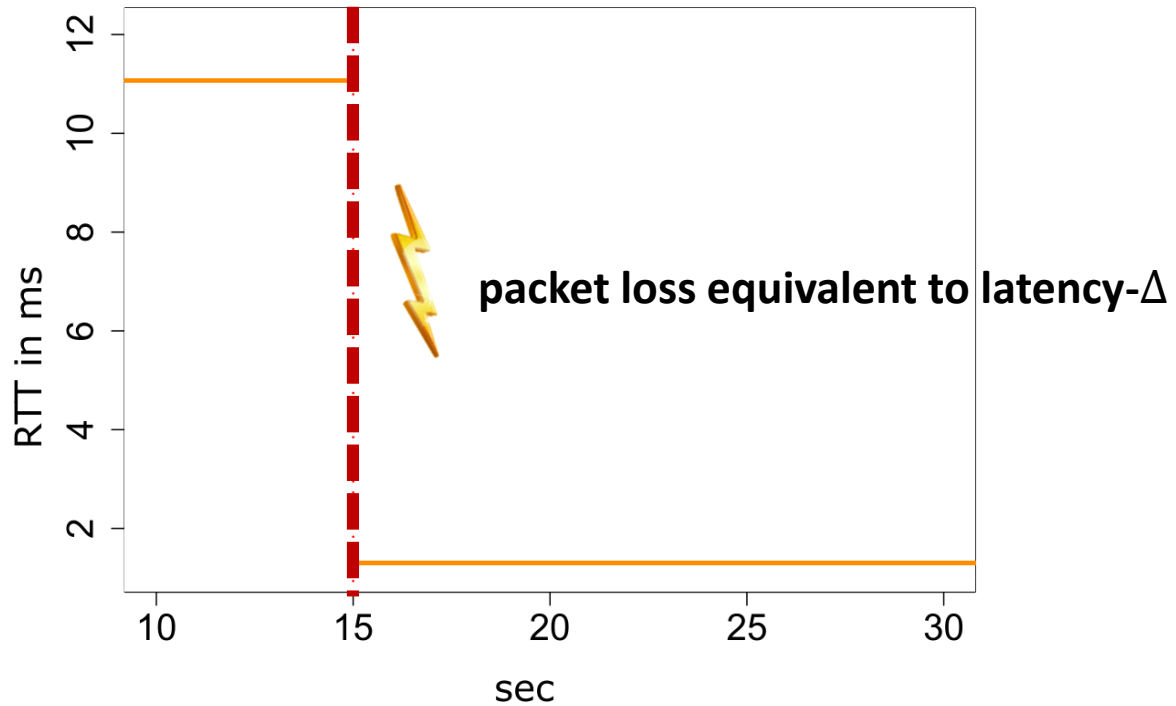
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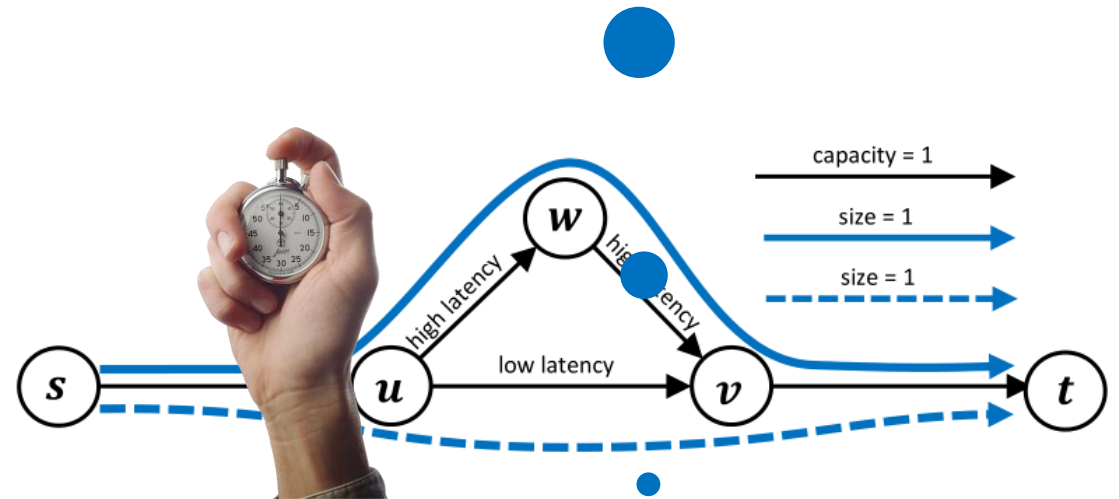
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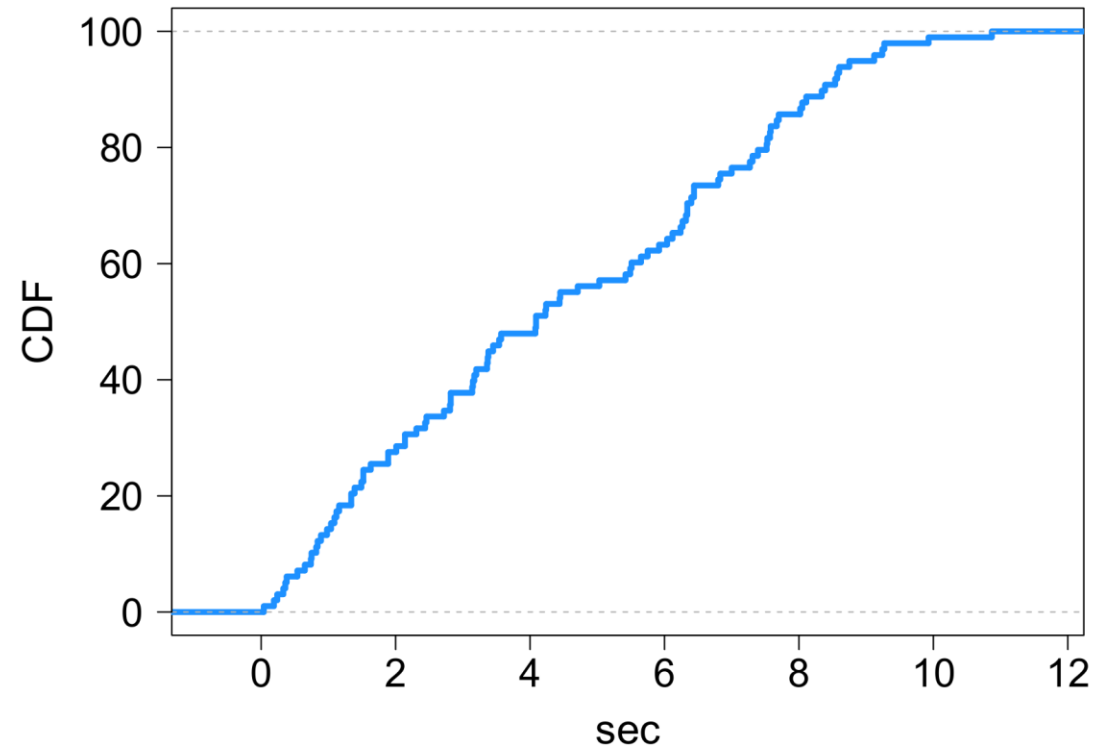
Because there is also work that focuses on better time synchronization, notably by Mizrahi et al. <https://sites.google.com/site/timedsdn/>



Even holds without asynchrony



## CDF of the Congestion Duration



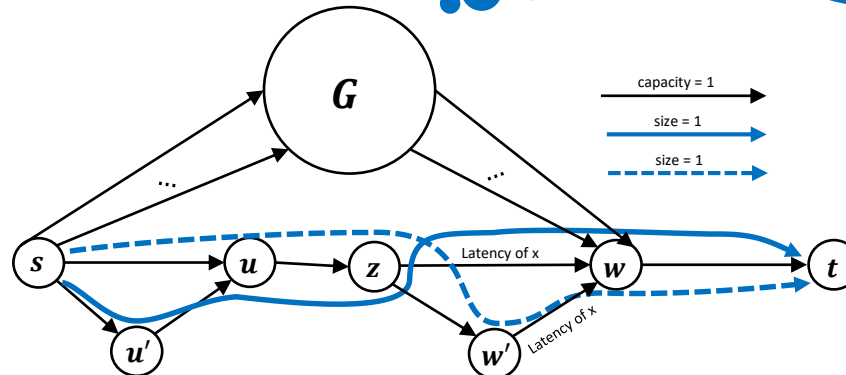
## Recap

- Common (coarse-grained) model:
  - Sum for all flows:  $\text{Max}(\text{old flow rules}, \text{new flow rules})$  does not violate capacity [SWAN, SIGCOMM'13]
  - Decidable in polynomial time [Brandt et al., INFOCOM'16]
    - For unsplittable flows: NP-hard already for 2 flows
- Does not capture congestion due to flows congesting themselves!
  - How hard?

## How hard?

- Unit latencies and splittable flow of unit size:
  - Already NP-hard for a single flow!

Find a temporary path to offload parts of the flow

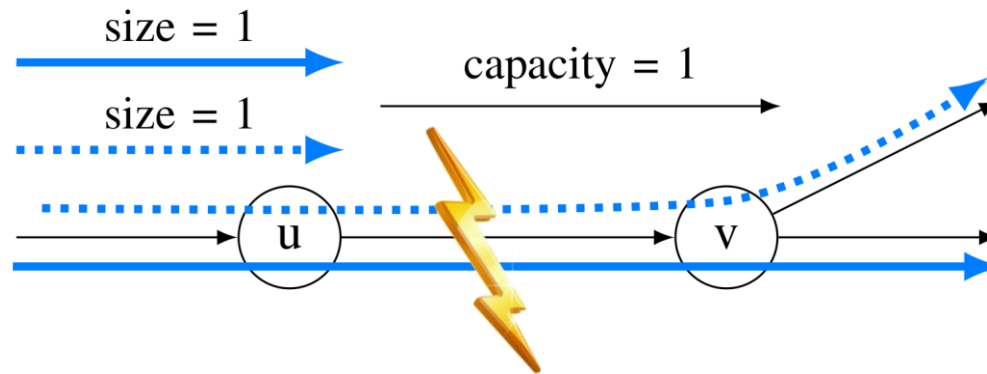


## Recap of the last few slides

- Common (coarse-grained) model:
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    - For unsplittable flows: NP-hard already for 2 flows
- Does not capture congestion due to flows congesting themselves!
  - How hard?
    - NP-hard for unit size/latency and splittable flows ☹️
- How to fix?
  - Treat old and new flow rules as separate flows?

## Old and New as Different Entities

- Idea: We can handle interplay between different flows
  - Handle old and new as different flows?
    - Prevents such congestion in popular approaches, eg SWAN, Dionysus, zUpdate etc.





**Relax And Take it Easy!**





## Relax for Polynomial-Time Lossless Updates

- Idea: Relax the problem formulation
  - Be congestion-free for *any* set of latencies
    - (I.e., adversary may change latencies at any time)
- Now congestion-free intermediate steps become **reversible**
- Rough structure of the algorithm (for splittable flows):
  - Take old (new) state, reach intermediate state where critical set of edges have spare capacity
    - Not possible? No congestion-free migration possible.



Achieved by spreading  
the network load

## Recap of the last few slides

- Common (coarse-grained) model:
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    - For unsplittable flows: NP-hard already for 2 flows
- Does not capture congestion due to flows congesting themselves!
  - NP-hard for unit size/latency and splittable flows ☹️
- By relaxing latency constraints:
  - Again polynomial-time decidable
- Interestingly: Augmenting flow idea still works **even without relaxing latency constraints!**

How to extend beyond  
a single destination?

But requires non-fixed  
new flow paths

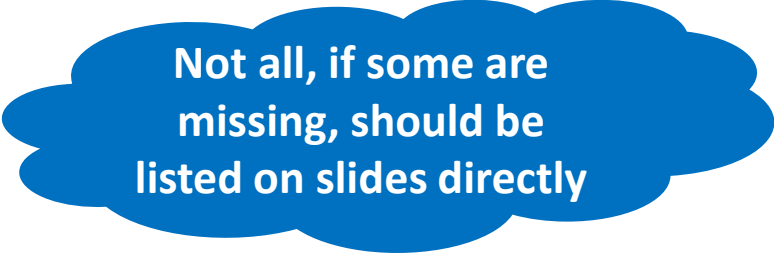



## Open Problems and Outlook in General

- Various algorithmic and complexity questions for a centralized controller
  - See recent survey
- First connections to more classic distributed computing topics are made
  - *Proof-labeling*
    - Very basic right now, how to build more complex/efficient systems?
- Maybe the bigger question: How to properly distribute the centralized controller
  - Opportunity: The SUPPORTED model / *preprocessing*

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Not all, if some are missing, should be listed on slides directly

# Central Control over Distributed Asynchronous Systems: A Tutorial on Software-Defined Networks and Consistent Network Updates

Klaus-T. Foerster

